PET ENGINEERING COLLEGE

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DEPARTMENT OF ELECTRONICS AND COMMUNICATION ENGINEERING

UNIT - I

CLASS	**: S3 ECE**
SUBJECT CODE	**: EC3354**
SUBJECT NAME	**: SIGNALS AND SYSTEMS**
REGULATION	**: 2021**

EC8352 - SIGNALS AND SYSTEMS

Unit - I : Classification Signals and Systems.

Standard signals - Step, Ramp, Pulse, Impulse, Real and Complex exponentials and sinusoids, classification of signals - Continuous Time (CT), Discrete Time (DT), Periodic, Aperiodic, Deterministic, Random, Energy, Power signals. classification of systems - CT systems and DT systems, Linear and Nonlinear system. Time variant and Time invariant system, Causal and Non causal, Stable and Unstable system.

Unit - II : Analysis of Continuous Time Signals.

Fourier series for periodic signals, Fourier Transform and its properties, Laplace Transform and it's properties.

Unit - III : Linear Time Invariant Continuous Time System

Impulse response - Convolution Integrals Differential equation - Fourier and Laplace transforms in analysis of CT systems. Systems connected in series and parallel.

Unit - IV : Analysis of Discrete Time Signals

Baseband Signal Sampling, Fourier Transform of Discrete time signal and it's properties, Z-transform and it's properties.

Unit - V : Linear Time Invariant Discrete Time System

Impulse response - Difference equations - Convolution sum - Discrete Fourier Transform and z transform analysis of Recursive and Non-recursive systems - DT systems connected in series and parallel.

TEXT BOOK :

1. Allan. V. Oppenheim, S. Wilsky and S.H. Nawab Signals and Systems, Pearson - 2015

REFERENCES :

1. B.P. Lathi, Principles of Linear System and Signals, 2nd Edition, Oxford 2009.

2. R.E. Zeimer, W.H. Tranter and R.D. Fannin Signals and Systems - Continuous and Discrete Pearson 2007.

3. John Alan Stuller An Introduction to Signals and Systems, Thomson 2007.

UNIT-I : CLASSIFICATION OF SIGNALS AND SYSTEMS

INTRODUCTION:

In this modern era of microelectronics, signals and systems play very vital roles. It is an extraordinary subject with diverse applications in areas of science and technology such as circuit design, seismology, communication, bio medical engineering, energy generation and distribution, speech processing, etc.,

SIGNALS:

- Signal is a "physical quantity varying with respect to one or more independent variables", which contains some information i.e.,

A signal may be a function of time, temperature, position, pressure, distance, etc.,

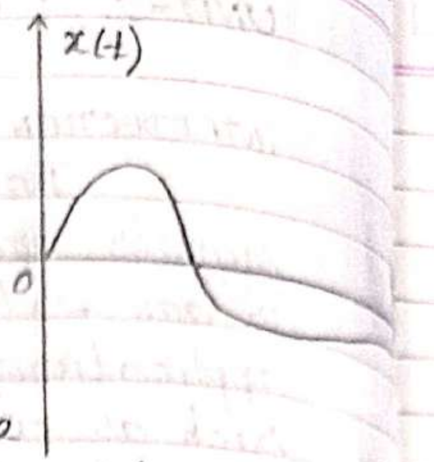
Examples:

- A telephone or a television signal
- Monthly sales of a corporation
- Daily closing prices of a stock market.
- An electrical charge is distributed over a body
- Music, speech, picture and video signals.

* Based upon their nature and characteristics in the time domain, the signal may be broadly classified as

1. Continuous Time (CT) Signals
2. Discrete Time (DT) Signals.

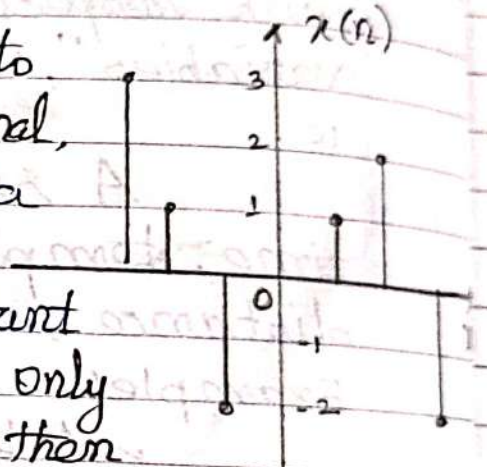
CONTINUOUS TIME SIGNALS



✓ A signal is said to be a Continuous Time (CT) signal. if and only if it will have a strength of some value of amplitude at every instant of time i.e., " A signal of continuous amplitude and time is called as Continuous time signal

Eg: ✓ A telephone or a television signal
- Music, speech, picture, audio and video signals.

DISCRETE TIME SIGNALS:



✓ A signal is said to be a Discrete Time (DT) signal, if and only if it will have a strength of some value or amplitude at certain time instant i.e., If the signal is defined only at discrete instant of time, then it is known as Discrete time (DT) signal.

Eg: ✓ Monthly sales of a corporation
✓ Quarterly Gross National Product (NGP)
✓ Stock market daily averages.

* Based upon on the property of a signal, it may be classified as follows

1. Periodic and Non periodic Signals
2. Even and odd Signals
3. Energy and Power Signals
4. Deterministic and Random Signals
5. Causal, Anti causal and Noncausal Signals

PERIODIC SIGNAL:

A signal $x(t)$ is said to be periodic if for some positive constant T, the smallest value of T that satisfies the periodicity condition

$$\boxed{x(t) = x(t+T)} \text{ for all 't'}$$

i.e., A signal which repeats itself after a fixed time period is called as a PERIODIC SIGNAL.

* T is also ~~known~~ called as Fundamental period and defined as the time taken for the signal to complete it's one cycle.

Eg: Sine, Cosine, Square waves.

NONPERIODIC SIGNAL:

A signal $x(t)$ is said to be nonperiodic, it does not satisfy the periodicity condition.

$$\boxed{x(t) \neq x(t+T)}$$

i.e., A signal which does not repeat itself after a fixed time period is called as a NONPERIODIC SIGNAL. Also known as Aperiodic Signal.

Eg: Exponential Signal.

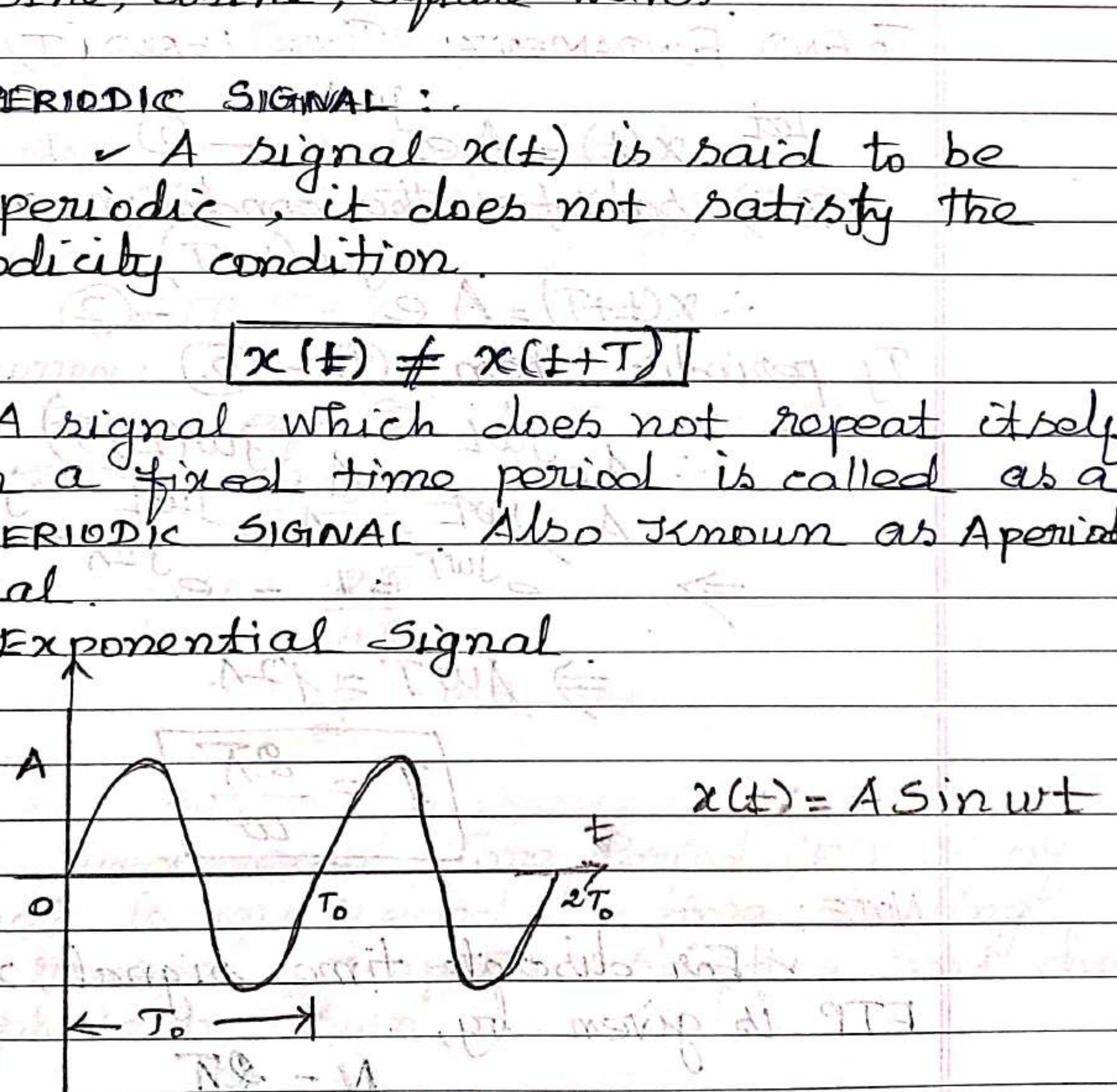


Fig. Periodic Signal.

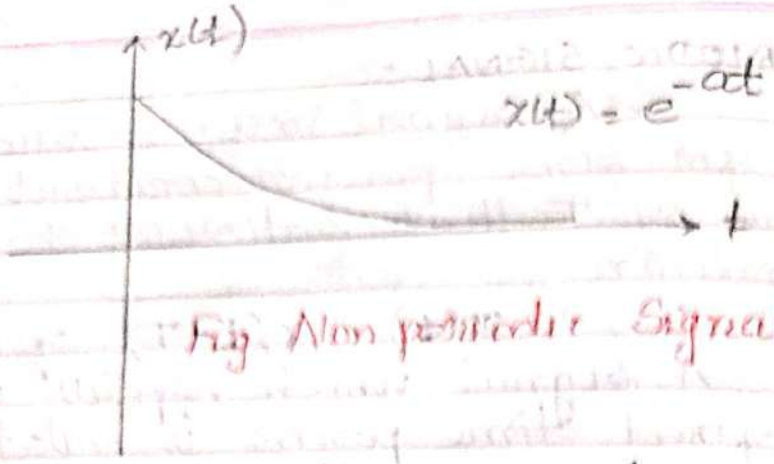


Fig. Non periodic Signal

NOTE:

* For discrete time signal periodic signal,

$$\boxed{x(n) = x(n+N)}$$

* For discrete time non periodic signal.

$$\boxed{x(n) \neq x(n+N)}$$

TO FIND FUNDAMENTAL TIME PERIOD (T)

Let $x(t) = Ae^{j\omega t}$ — ①

$x(t)$ satisfies the condition $x(t) = x(t+T)$

$\therefore x(t+T) = A e^{j\omega(t+T)}$ — ②

If periodic then ① = ② $\quad \therefore a^{m+n} = a^m . a^n$

$$\therefore Ae^{j\omega t} = A e^{j\omega(t+T)}$$

$$\cancel{Ae^{j\omega t}} = \cancel{A e^{j\omega t}} e^{j\omega T}$$

$$\Rightarrow e^{j\omega T} = 1 = e^{j2\pi}$$

$$\Rightarrow \cancel{j}\omega T = \cancel{j}2\pi$$

$$\boxed{T = \frac{2\pi}{\omega}}$$

NOTE:

For discrete time signal, $x(n)$, the FTP is given by,

$$N = \frac{2\pi}{\Omega}$$

Problems 01: Find the FTP (Fundamental Time Period) of following signals.

(i) $x_1(t) = 3\sin 2\pi t$ (ii) $x_2(t) = 5\sin(4\pi t + \pi/6)$

iii) $x_3(t) = \sin^2(4\pi t)$

i) GIVEN: $A \sin \omega t$

$$x_1(t) = 3\sin 2\pi t \quad -①$$

TO FIND:

$$FTP, T = ?$$

SOLUTION:

From eqn ① $A = 3$; $\omega = 2\pi$.

W.K.T $T = \frac{2\pi}{\omega}$

$$\therefore T = \frac{2\pi}{2\pi} = 1$$

$$T = 1 \text{ sec.}$$

ii) GIVEN: $A \sin(\omega t + \theta)$

$$x_2(t) = 5\sin(4\pi t + \pi/6) \quad -②$$

TO FIND:

$$FTP, T = ?$$

SOLUTION:

From eqn ②, $A = 5$; $\omega = 4\pi$

W.K.T $T = \frac{2\pi}{\omega}$

$$\therefore T = \frac{2\pi}{4\pi}$$

$$T = \frac{1}{2} = 0.5 \text{ sec.}$$

NOTE:-

Fundamental Time Period (FTP), T of signal is unaffected by time shifting, time reversal, amplitute shifting and change in phase of signal.

GIVEN: $\because \sin^2\theta = \frac{1-\cos 2\theta}{2}$

iii) $x_3(t) = \sin^2(4\pi t)$

$x_3(t) = \frac{1-\cos 8\pi t}{2}$

$x_3(t) = \frac{1}{2} - \frac{1}{2}\cos 8\pi t$ —— ③

$A_1 + A_2 \cos \omega t$

TO FIND:

FTP, $T = ?$

SOLUTION:

From eqn ③ $\omega = 8\pi$

W.K.T $T = \frac{2\pi}{\omega}$

$\therefore T = \frac{2\pi}{8\pi}$

$T = \frac{1}{4}$ sec.

NOTE:

The sum of two (or) more than two periodic signal will be periodic if ratio of there FTP are rational number.

i.e., $x(t) = x_1(t) + x_2(t)$

$$\begin{array}{cc} \downarrow & \downarrow \\ T_1 & T_2 \end{array}$$

$\frac{T_1}{T_2}$ = Rational number

Problem 02: Find FTP of if it is periodic.

(i) $x_1(t) = \sin 2t + \cos 3\pi t$

(ii) $x_2(t) = \sin 2\pi t + \cos \sqrt{2}\pi t$

(iii) $x_3(t) = \sin 4\pi t + \sin 7\pi t$

i) GIVEN: $\sin \omega_1 t + \cos \omega_2 t$

$x_1(t) = \sin 2t + \cos 3\pi t$ —— ①

TO FIND:

~~FTP, T~~ ? Periodic (or) Not

SOLUTION:

From eqn ① $\omega_1 = 2$; $\omega_2 = 3\pi$

W.K.T $T_1 = \frac{2\pi}{\omega_1}$ & $T_2 = \frac{2\pi}{\omega_2}$

$\therefore T_1 = \frac{2\pi}{2}$; $T_2 = \frac{2\pi}{3\pi}$.

$\Rightarrow \frac{T_1}{T_2} = \frac{\pi}{(2/3)} = \frac{3\pi}{2}$ = Irrational No.

$\therefore$ The given signal $x(t) = \sin 2t + \cos 3\pi t$ is a Non periodic signal.

ii) GIVEN: $\sin \omega_1 t + \cos \omega_2 t$

$x_2(t) = \sin 2\pi t + \cos \sqrt{2}\pi t$ — ②

TO FIND:

Periodic (or) Not

SOLUTION:

From eqn ① $\omega_1 = 2\pi$; $\omega_2 = \sqrt{2}\pi$

W.K.T

$T_1 = \frac{2\pi}{\omega_1}$; $T_2 = \frac{2\pi}{\omega_2}$

$\therefore T_1 = \frac{2\pi}{2\pi} = 1$; $T_2 = \frac{2\pi}{\sqrt{2}\pi} = \sqrt{2}$.

$\Rightarrow \frac{T_1}{T_2} = \frac{1}{\sqrt{2}}$ = Irrational No.

$\therefore$ The given signal $x(t) = \sin 2\pi t + \cos\sqrt{2}\pi t$ is a Non-periodic signal.

iii) GIVEN:

$x_3(t) = \sin 4\pi t + \sin 7\pi t$ — ③

TO FIND:

Periodic or Not

SOLUTION:

From eqn ③ $\omega_1 = 4\pi$; $\omega_2 = 7\pi$

W.K.T $T_1 = \frac{2\pi}{\omega_1}$; $T_2 = \frac{2\pi}{\omega_2}$

$\therefore T_1 = \frac{2\pi}{4\pi} = \frac{1}{2}$; $T_2 = \frac{2\cancel{\pi}}{7\cancel{\pi}} = \frac{2}{7}$.

$\therefore \frac{T_1}{T_2} = \frac{(1/2)}{(2/7)} = \frac{1}{2} \times \frac{7}{2} = \frac{7}{4}$

= Rational No.

To Find FTP,

$T = \frac{T_1}{T_2} = \frac{7}{4}$

$\Rightarrow T_1 = \frac{7}{4}(T_2) = \frac{7}{4}\left(\frac{2}{7}\right) = 1/2$

$\boxed{T = 1/2}$

ENERGY AND POWER SIGNALS:

✓ An ENERGY signal is one which has finite energy and zero power. Hence $x(t)$ is an energy signal, if:

$$\boxed{0 < E < \infty \; ; \; P = 0}$$

✓ The POWER signal is one which has finite power and infinite energy. Hence $x(t)$ is a power signal, if:

$$\boxed{0 < P < \infty \; ; \; E = \infty}$$

✓ If the signal does not satisfy any of the above two coditions, the it is neither an energy nor a power signal.

FOR CT signal, $x(t)$:

$$\text{Energy, } E = \int_{-\infty}^{\infty} |x(t)|^2 \, dt$$

$$\text{Power, } P = \lim_{T \to \infty} \frac{1}{2T} \int_{-T}^{T} |x(t)|^2 \, dt$$

FOR DT SIGNAL, $x(n)$:

$$\text{Energy, } E = \sum_{n=-\infty}^{\infty} |x(n)|^2$$

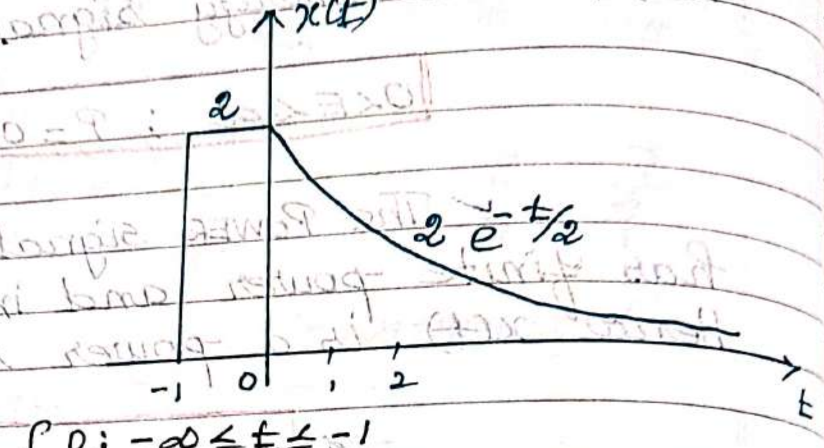
$$\text{Power, } P = \lim_{N \to \infty} \frac{1}{2N+1} \sum_{n=-N}^{N} |x(n)|^2$$

NOTE:

* Non periodic signals are energy signals
  Eg: A single rectangular pulse.

* Periodic signals are power signals.
  Eg: Periodic pulse signal.

Problem: 1 Fig shows the signal $x(t)$. Determine whether the signal is an energy or a power signal or neither.



GIVEN:

$$x(t) = \begin{cases} 0; & -\infty \le t \le -1 \\ 2; & -1 \le t \le 0 \\ 2e^{-t/2}; & 0 \le t \le \infty \end{cases}$$

TO FIND:

Energy, E & Power, P

SOLUTION:

W.K.T Energy, $E = \int_{-\infty}^{\infty} |x(t)|^2 dt$

$$E = \int_{-\infty}^{-1} |x(t)|^2 dt + \int_{-1}^{0} |x(t)|^2 dt + \int_{0}^{\infty} |x(t)|^2 dt$$

$$= \int_{-1}^{0} (2)^2 dt + \int_{0}^{\infty} \left[2e^{-t/2}\right]^2 dt$$

$$= 4\left[t\right]_{-1}^{0} + 4\left[-e^{-t}\right]_{0}^{\infty}$$

$$= 4\left[0+1\right] + 4\left[-e^{-\infty} + e^{0}\right]$$

$$= 4+4 = 8$$

Energy, $E = 8$

$\therefore$ Power $= 0$

$\therefore$ The given signal $x(t)$ is an energy signal.

Problem 2 Find the energy and power of following sequences.

(i) $x_1(n) = a^n u(n)$

(ii) $x_2(n) = u(n)$

(i) GIVEN:

$$x_1(n) = a^n u(n) = a^n \ ; \ 0 \leq n \leq \infty$$

$\because u(n)$ = Unit step signal $= \begin{cases} 1 ; & n \geq 0 \\ 0 ; & n < 0. \end{cases}$

TO FIND:

Energy, E & Power, P

SOLUTION:

W.K.T $E = \sum_{n=-\infty}^{\infty} |x(n)|^2$

$$= \sum_{n=-\infty}^{\infty} |a^n u(n)|^2$$

$$= \sum_{n=0}^{\infty} a^{2n}$$

$$= \sum_{n=0}^{\infty} (a^2)^n$$

$$\sum_{n=0}^{\infty} a^n = \frac{1}{1-a} \text{ if } a \neq 1$$

$$\boxed{E = \frac{1}{1-a^2}}$$

$\therefore$ Power, $P = 0$

$\therefore$ The signal $x_1(n)$ is an Energy signal.

Eg: $x(n) = (1/3)^n u(n)$. Find Energy and Power.

GIVEN:

$$x(n) = (1/3)^n u(n) = (1/3)^n \ ; \ 0 \leq n \leq \infty$$

TO FIND:

Energy, $E = ?$; Power, $P = ?$

SOLUTION:

$$\text{Energy, } E = \sum_{n=-\infty}^{\infty} |x(n)|^2$$

$$\therefore E = \sum_{n=-\infty}^{\infty} \left| (1/3)^n u(n) \right|^2$$

$$= \sum_{n=0}^{\infty} \left[(1/3)^n \right]^2$$

$$= \sum_{n=0}^{\infty} \left[(1/3)^2 \right]^n$$

$$= \sum_{n=0}^{\infty} (1/9)^n \qquad \sum_{n=0}^{\infty} a^n = \frac{1}{1-a}$$

$$= \frac{1}{1 - 1/9}$$

$$\boxed{E = 9/8}$$

$$\therefore \boxed{P = 0}$$

) GIVEN:

$$x_2(n) = u(n) = 1; \quad n \geq 0$$
$$\qquad\qquad\quad 0; \quad n < 0$$

TO FIND:

Energy, $E = ?$; Power, $P = ?$

SOLUTION:

$$\text{Energy, } E = \sum_{n=-\infty}^{\infty} |x(n)|^2$$

$$\therefore E = \sum_{n=-\infty}^{\infty} |u(n)|^2$$

$$= \sum_{n=-\infty}^{-1} |u(n)|^2 + \sum_{n=0}^{\infty} |u(n)|^2$$

$$= \sum_{n=0}^{\infty} (1)^n$$

$$\boxed{E = \infty}$$

$$\text{Power, } P = \underset{N\to\infty}{Lt} \frac{1}{2N+1} \sum_{n=-N}^{N} |x(n)|^2$$

$$\therefore P = \underset{N\to\infty}{Lt} \frac{1}{2N+1} \sum_{n=-N}^{N} |u(n)|^2$$

$$= \underset{N\to\infty}{Lt} \frac{1}{2N+1} \left\{ \sum_{n=-N}^{-1} |u(n)|^2 + \sum_{n=0}^{N} |u(n)|^2 \right\}$$

$$= \underset{N\to\infty}{Lt} \frac{1}{2N+1} \sum_{n=0}^{N} (1)^2$$

$$= \underset{N\to\infty}{Lt} \frac{1}{2N+1} \sum_{n=0}^{N} (1)^n \qquad \because \sum_{n=0}^{N} (1)^n = N+1$$

$$= \underset{N\to\infty}{Lt} \frac{1}{2N+1} N+1$$

$$= \underset{N\to\infty}{Lt} \frac{\cancel{N}(1+1/N)}{\cancel{N}(2+1/N)}$$

$$= \frac{(1+1/\infty)}{(2+1/\infty)} \qquad \because \frac{1}{\infty} = 0$$

$$\boxed{P = 1/2}$$

$\therefore$ The given signal $x_2(n) = u(n)$ is a power signal.

Problem 3: Find Energy and power of ramp signal.

GIVEN:

Unit Ramp signal ; $r(t) = t \; ; \; t \geq 0$

$\qquad\qquad\qquad\qquad 0 \; ; \; t < 0.$

TO FIND:

Energy, $E = ?$; Power, $P = ?$

SOLUTION:

$$\text{Energy, } E = \int_{-\infty}^{\infty} |x(t)|^2 dt$$

$$\therefore E = \int_{-\infty}^{\infty} |r(t)|^2 dt$$

$$= \int_{0}^{\infty} (t)^2 dt$$

$$= \left[\frac{t^3}{3}\right]_0^{\infty}$$

$$\boxed{E = \infty}$$

$$\text{Power, } P = \lim_{T\to\infty} \frac{1}{T} \int_{-T}^{T} |x(t)|^2 dt$$

$$\therefore P = \lim_{T\to\infty} \frac{1}{T} \int_{-T}^{T} |r(t)|^2 dt$$

$$= \lim_{T\to\infty} \frac{1}{T} \int_{0}^{T} (t)^2 dt$$

$$= \lim_{T\to\infty} \frac{1}{T} \left[\frac{t^3}{3}\right]_0^{T}$$

$$= \lim_{T\to\infty} \frac{1}{T} \left[\frac{T^3}{3}\right]$$

$$\boxed{P = 0}$$

$\therefore$ The given signal, $r(t)$ is either Energy nor power

STANDARD SIGNALS :

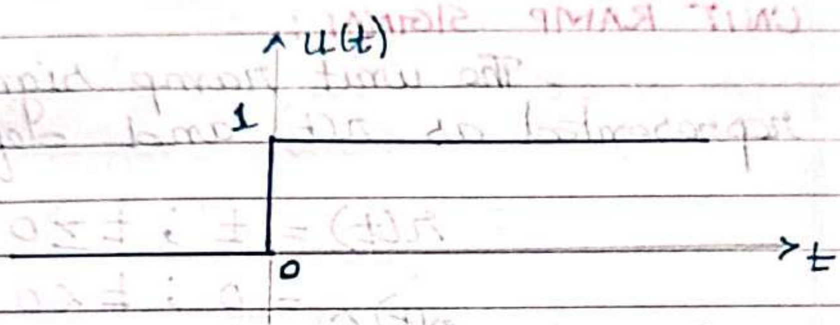
Unit step, Unit impulse, Unit ramp, pulse, real and complex exponential, sinusoidal signals are known as Standard signals (or) Test signals (or) Basic Elementary signals.

UNIT STEP SIGNALS :

Unit step signal can be represented as u(t) and defined as,

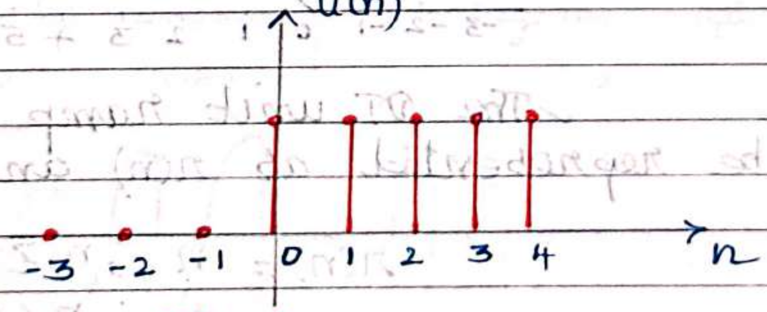
FOR CT

$$u(t) = \begin{cases} 1 ; & t \geq 0 \\ 0 ; & t < 0 \end{cases}$$



FOR DT DT unit step signal can be represented as u(n) and defined as,

$$u(n) = \begin{cases} 1 ; & n \geq 0 \\ 0 ; & n < 0 \end{cases}$$



UNIT IMPULSE SIGNAL :

Unit impulse signal can be represented as δ(t) and defined as.

FOR CT

$$\delta(t) = \begin{cases} 1 ; & t = 0 \\ 0 ; & t \neq 0 \end{cases}$$

FOR DT ✓ DT unit impulse signal can be represented as $\delta(n)$ and defined as,

$$\delta(n) = \begin{cases} 1 \; ; & n \geq 0 \\ 0 \; ; & n < 0 \end{cases}$$

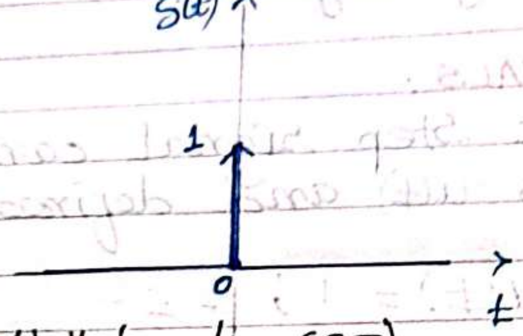


Fig. Unit Impulse (CT)

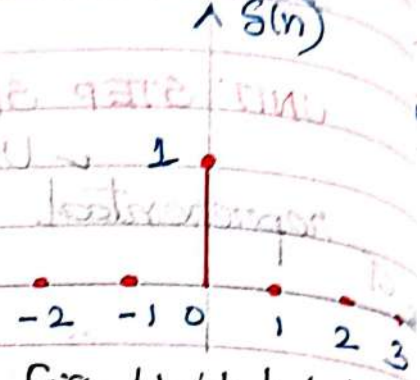
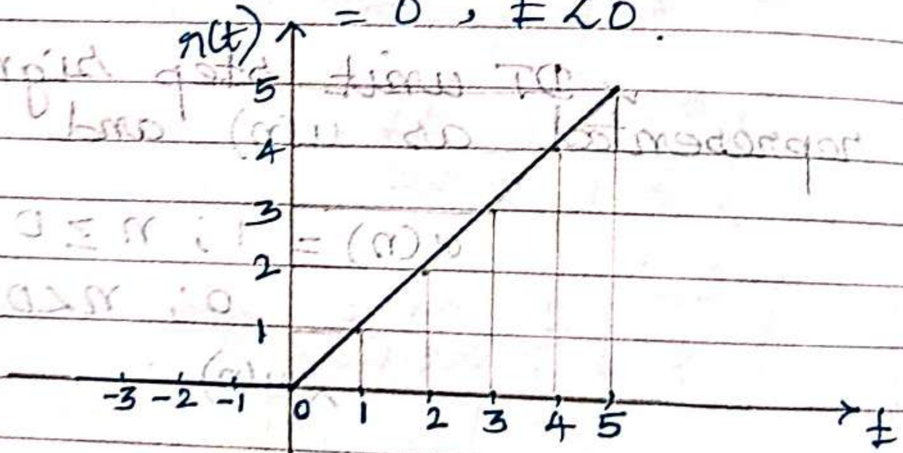


Fig. Unit Impulse (DT)

UNIT RAMP SIGNAL:

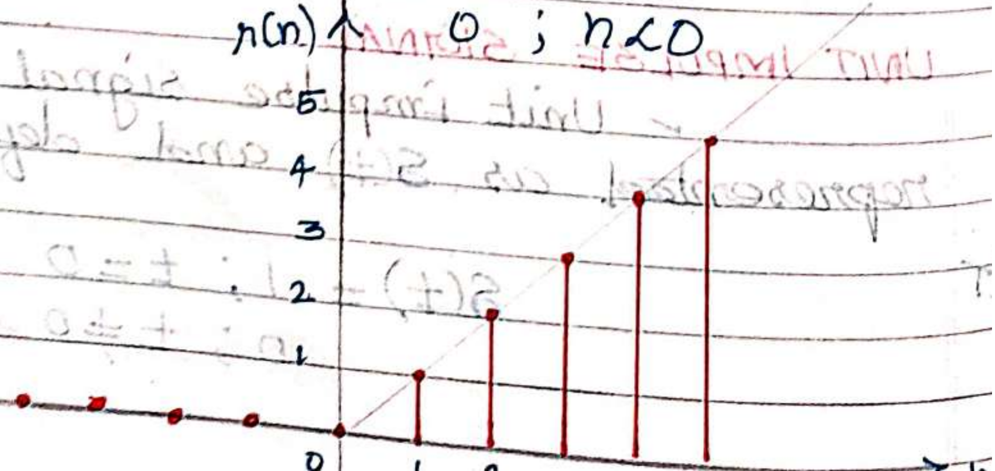
FOR CT ✓ The unit ramp signal can be represented as $r(t)$ and defined as.

$$r(t) = \begin{cases} t \; ; & t \geq 0 \\ 0 \; ; & t < 0 \end{cases}$$



FOR DT ✓ The DT unit ramp signal can be represented as $r(n)$ and defined

$$r(n) = \begin{cases} n \; ; & n \geq 0 \\ 0 \; ; & n < 0 \end{cases}$$



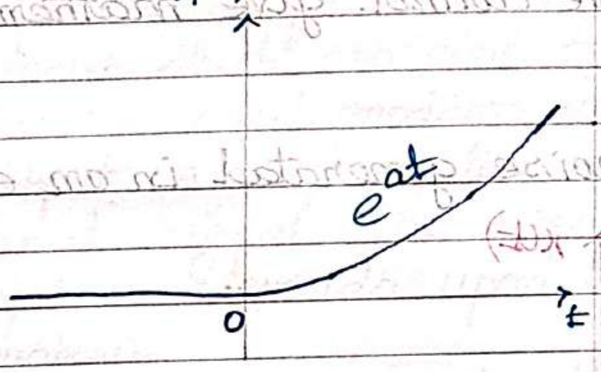
REAL EXPONENTIAL SIGNALS :

Rising Real Exponential | **Decaying Real Exponential**

FOR CT

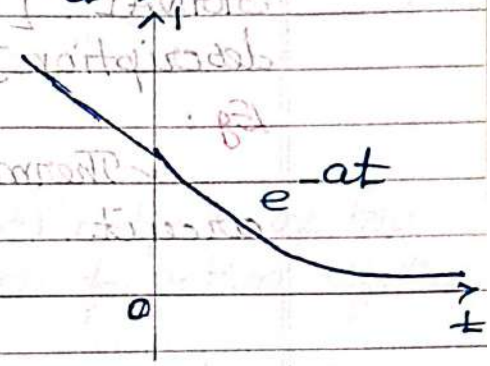
- It is defined as,

$$e_r(t) = e^{at}$$



- It is defined as,

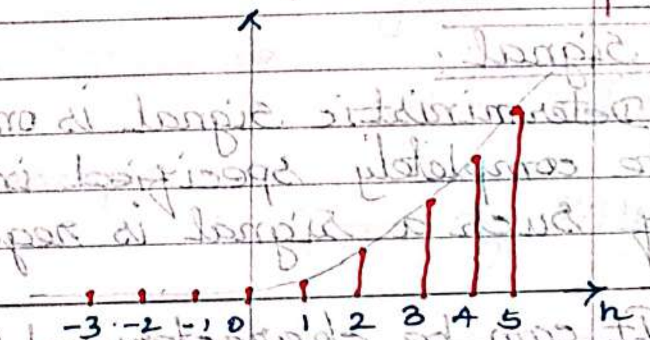
$$e_d(t) = e^{-at}$$



FOR DT

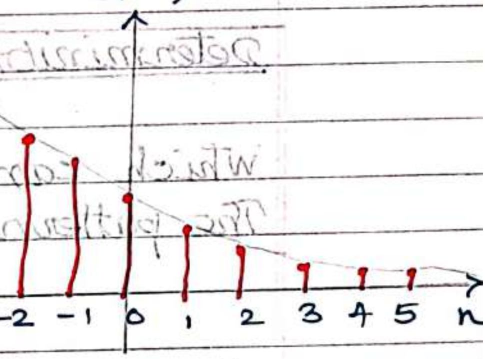
- It is defined as

$$e_r(n) = e^{an}$$



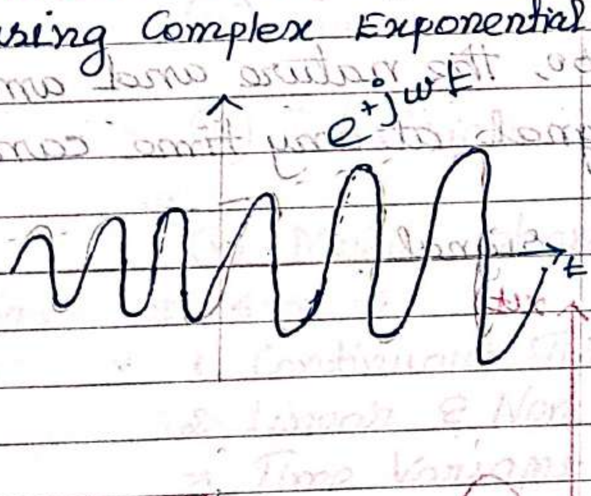
- It is defined as,

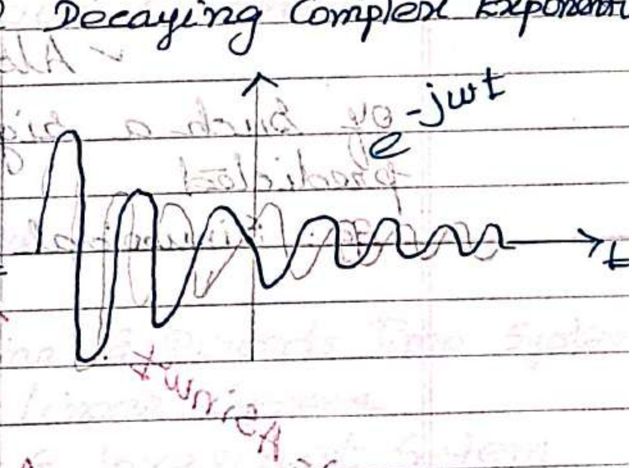
$$e_d(n) = e^{an}$$



COMPLEX EXPONENTIAL SIGNALS :

Rising Complex Exponential | **Decaying Complex Exponential**



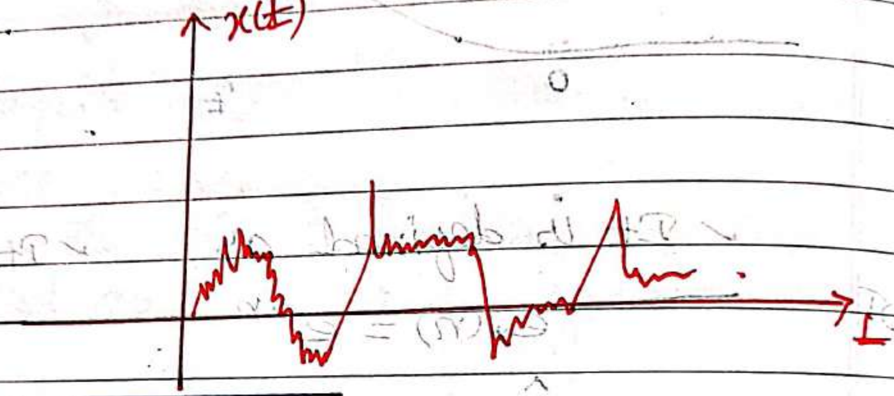


Random Signals:

✓ Random Signal is one whose occurrence is always in nature. The pattern of such a signal is quite irregular.

✓ Also known as "NON DETERMINISTIC SIGNAL". [ie, We cannot give mathematical description].

Eg: ✓ Thermal noise generated in an electric circuit.



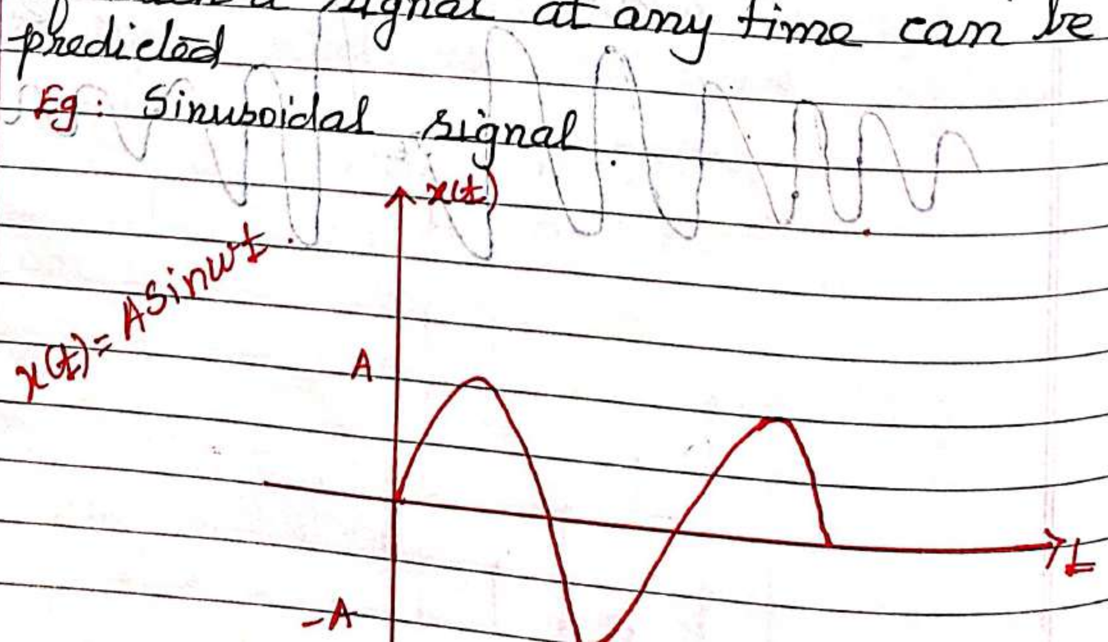
Deterministic Signal:

✓ Deterministic signal is one which can be completely specified in time. The pattern of such a signal is regular.

✓ It can be characterized (or) defined mathematically.

✓ Also, the nature and amplitude of such a signal at any time can be predicted

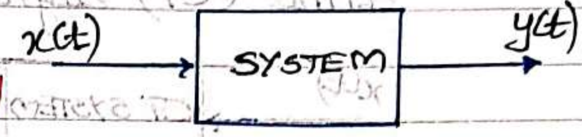
Eg: Sinusoidal signal



SYSTEMS:

- A system is a physical device which operates on input signal and produces an output. i.e.,

A system may be defined as a set of elements (or) functional blocks which are connected together and produces an output in response to an input signal. The response (or) output of the system depends upon transfer function of the system.



- Mathematically, the functional relationship between input and output may be written as

$$y(t) = f[x(t)]$$

OR

$$y(t) = T[x(t)]$$

Examples:

(i) Types of Filters

(ii) Amplifier

(iii) Communication Channel

(iv) Television

(v) Mobile phone

TYPES OF SYSTEM:

1. Continuous Time & Discrete Time System
2. Linear & Non Linear System
3. Time Variant & Invariant System
4. Static & Dynamic System
5. Causal & Non causal System
6. Stable & Unstable System

CONTINOUS TIME SYSTEM

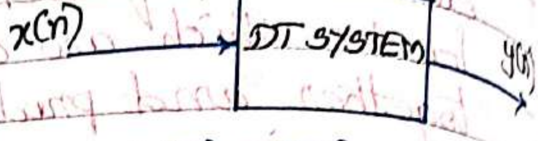
The input and output of a system are both continuous time (CT) signals.

$$y(t) = T[x(t)]$$

Eg: i) Audi-Video amplifier
ii) Power Supply Unit
iii) RLC Circuit

DISCRETE TIME SYSTEM

The input and output of a system are both discrete time (DT) signals.



$$y(n) = T[x(n)]$$

Eg: i) Microprocessors
ii) Semi conductor Memory
iii) Shift Registers.

LINEAR SYSTEMS AND NON LINEAR SYSTEMS:

A system is linear if it satisfies linearity property (or) the superposition principles. i.e.,

→ A linear system follows the law of superposition

→ This law is necessary and sufficient to prove linearity of system.

* It is a combination of two laws:
(i) law of additivity
(ii) law of Homogenity

Mathematically,

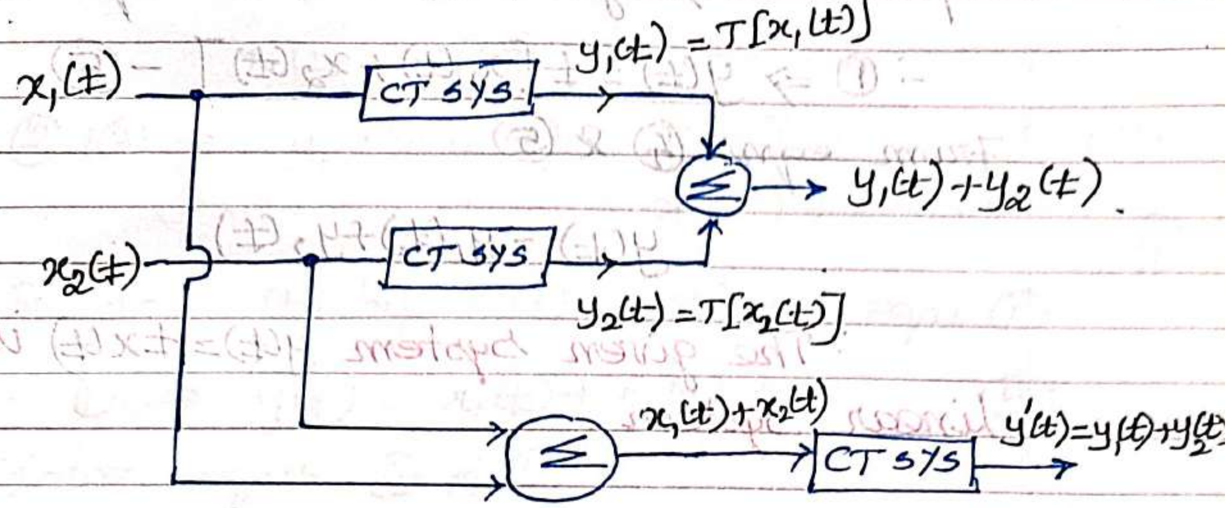
FOR CT

$$T[a_1x_1(t) + a_2x_2(t)] = a_1T[x_1(t)] + a_2T[x_2(t)]$$
$$= a_1y_1(t) + a_2y_2(t)$$

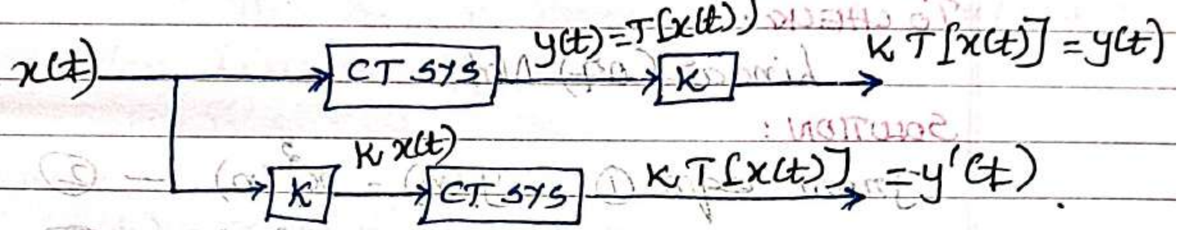
FOR DT

$$T[a_1x_1(n) + a_2x_2(n)] = a_1T[x_1(n)] + a_2T[x_2(n)]$$
$$= a_1y_1(n) + a_2y_2(n).$$

LAW OF ADDITIVITY:



LAW OF HOMOGENITY:



Problem: Check whether the following systems are linear or not

a) $y(t) = t\,x(t)$ b) $y(n) = x^2(n)$

c) $y(t) = x(t) + 10$ d) $y(t) = x[\sin t]$ (H.W)

e) $y(n) = \sin[x(n)]$ (H.W) f) $y(t) = e^{x(t)}$ (H.W)

a) **GIVEN:**

$$y(t) = t\,x(t) \quad — ①$$

TO CHECK:

Linear (or) Not

SOLUTION:

From eqn ① $\quad y_1(t) = t\,x_1(t) \quad — ②$

$$y_2(t) = t\,x_2(t) \quad — ③$$

② + ③ $\quad y_1(t) + y_2(t) = t\,x_1(t) + t\,x_2(t)$

$$= t[x_1(t) + x_2(t)] \quad — ④$$

Replace $x(t)$ by $x_1(t)+x_2(t)$ in eqn ①

$$\therefore ① \Rightarrow y(t) = t\left[x_1(t)+x_2(t)\right] \quad —⑤$$

From eqns ④ & ⑤

$$y(t) = y_1(t)+y_2(t)$$

$\therefore$ The given system $y(t) = t\,x(t)$ is a Linear System.

b) GIVEN:

$$y(n) = x^2(n) = [x(n)]^2 \quad —①$$

TO CHECK:

Linear (or) Not

SOLUTION:

From eqn ① $\quad y_1(n) = x_1^2(n) \quad —②$

$$y_2(n) = x_2^2(n) \quad —③$$

$$②+③ \Rightarrow y_1(n)+y_2(n) = x_1^2(n)+x_2^2(n) \quad —④$$

Replace $x(n)$ by $x_1(n)+x_2(n)$ in eqn ①

$$\therefore ① \Rightarrow y(n) = \left[x_1(n)+x_2(n)\right]^2$$

$$= x_1^2(n)+x_2^2(n)+2\,x_1(n)\,x_2(n) \quad —⑤$$

From eqns ④ & ⑤

$$y(n) \neq y_1(n)+y_2(n)$$

$\therefore$ The given system $y(n) = x^2(n)$ is a Non Linear system.

c) GIVEN:

$$y(t) = x(t)+10 \quad —①$$

TO CHECK:

Linear (or) Not

SOLUTION:

From eqn ①

$$y_1(t) = x_1(t) + 10 \quad —— ②$$

$$y_2(t) = x_2(t) + 10 \quad —— ③$$

$$② + ③ \Rightarrow y_1(t) + y_2(t) = x_1(t) + 10 + x_2(t) + 10$$

$$= x_1(t) + x_2(t) + 20 \quad —— ④$$

Replace $x(t)$ by $x_1(t) + x_2(t)$ in eqn ①

$$\therefore ① \Rightarrow y(t) = x_1(t) + x_2(t) + 10 \quad —— ⑤$$

From eqns ④ & ⑤

$$y(t) \neq y_1(t) + y_2(t).$$

$\therefore$ The given system $y(t) = x(t) + 10$ is a Non-linear system.

d) GIVEN:

$$y(t) = x[\sin t] \quad —— ①$$

TO CHECK:

Linear (or) Not.

SOLUTION:

From eqn ① $y_1(t) = x_1(\sin t)$ —— ②

$$y_2(t) = x_2(\sin t) \quad —— ③$$

$$② + ③ \Rightarrow y_1(t) + y_2(t) = x_1(\sin t) + x_2(\sin t) \quad —— ④$$

Replace $x(t)$ by $x_1(t) + x_2(t)$ in eqn ①

$$\therefore ① \Rightarrow y(t) = x_1(\sin t) + x_2(\sin t) \quad —— ⑤$$

From eqns ④ & ⑤

$$y(t) = y_1(t) + y_2(t)$$

$\therefore$ The given system $y(t) = x(\sin t)$ is a linear system.

e) GIVEN:

$$y(n) = \sin[x(n)] \quad \text{--- } (1)$$

TO CHECK:

Linear (or) Not

SOLUTION:

From eqn (1) $y_1(n) = \sin[x_1(n)]$ --- (2)

$y_2(n) = \sin[x_2(n)]$ --- (3)

(2)+(3) $\Rightarrow$ $y_1(n) + y_2(n) = \sin[x_1(n)] + \sin[x_2(n)]$ (4)

Replace $x(n)$ by $x_1(n) + x_2(n)$ in eqn (1)

$\therefore$ (1) $\Rightarrow$ $y(n) = \sin[x_1(n) + x_2(n)]$ --- (5)

From eqns (4) & (5)

$$y(n) \neq y_1(n) + y_2(n)$$

$\therefore$ The given system $y(n) = \sin[x(n)]$ is Non-linear system.

f) GIVEN:

$$y(t) = e^{x(t)} \quad \text{--- } (1)$$

TO CHECK:

Linear (or) Not

SOLUTION:

From eqn (1) $y_1(t) = e^{x_1(t)}$ --- (2)

$y_2(t) = e^{x_2(t)}$ --- (3)

(2)+(3) $\Rightarrow$ $y_1(t) + y_2(t) = e^{x_1(t)} + e^{x_2(t)}$ --- (4)

Replace $x(t)$ by $x_1(t) + x_2(t)$ in eqn (1)

$\therefore$ (1) $\Rightarrow$ $y(t) = e^{x_1(t) + x_2(t)}$ --- (5)

From eqns (4) & (5)

$$y(t) \neq y_1(t) + y_2(t)$$

$\therefore$ The given system $y(t) = e^{x(t)}$ is Non-linear system.

TIME INVARIANT AND TIME VARIANT SYSTEMS:

✓ A system is time variant if the input-output relationship ~~does not~~ vary with time i.e.,

A system is TIME VARIANT if its behaviour and input-output characteristics varying / changing w.r.to time.

Mathematically,

FOR CT If $y(t) = T[x(t)]$

then $T[x(t-T_0)] \neq y(t-T_0)$

FOR DT If $y(n) = T[x(n)]$

then $T[x(n-N_0)] \neq y(n-N_0)$

Eg:

✓ RC circuits.

✓ RLC circuits.

✓ A system is time invariant if the input-output relationship does not vary with time i.e.,

A system is TIME INVARIANT if its behaviour and input-output characteristics does not vary / change w.r.to time.

Mathematically,

FOR CT If $y(t) \neq T[x(t)]$

then $T[x(t-T_0)] = y(t-T_0)$

FOR DT If $y(n) = T[x(n)]$

Then $T[x(n-N_0)] = y(n-N_0)$

Eg:-

✓ Differentiator

Problem: Check whether the following systems are time-variant or not.

a) $y(t) = \sin x(t)$ b) $y(n) = n x(n)$

a) **GIVEN:**

$$y(t) = \sin x(t) \quad \text{--- (1)}$$

TO CHECK:

Time variant (or) Not

SOLUTION:

Replace 't' by $(t-T_0)$ in eqn (1) on i/p alone

$$\therefore (1) \Rightarrow y(t) = \sin[x(t-T_0)] \quad \text{--- (2)}$$

Replace 't' by $(t-T_0)$ in eqn (1)

$$\therefore (1) \Rightarrow y(t-T_0) = \sin[x(t-T_0)] \quad \text{--- (3)}$$

From eqns (2) & (3)

$$y(t) = y(t-T_0)$$

$\therefore$ The given system $y(t) = \sin x(t)$ is a Time invariant.

b) **GIVEN:**

$$y(n) = n x(n) \quad \text{--- (1)}$$

TO CHECK:

Time variant (or) Not

SOLUTION:

Replace 'n' by $(n-N_0)$ in eqn (1) on i/p alone

$$\therefore (1) \Rightarrow y(n) = n x(n-N_0) \quad \text{--- (2)}$$

Replace 'n' by $(n-N_0)$ in eqn (1)

$$\therefore (1) \Rightarrow y(n-N_0) = (n-N_0)\, x(n-N_0) \quad \text{--- (3)}$$

From eqns (2) & (3)

$$y(n) \neq y(n-N_0)$$

$\therefore$ The given system $y(n) = n x(n)$ is a Time variant.

CAUSAL AND NONCAUSAL SYSTEM:

✓ If the output of system depends only on present and past values of input but does not depend on future values of input then the system is called Causal system i.e.,

If output of system is independent of future values of input at each and every instant of time then system will be causal.

NOTE:

✓ Causal systems are physically realisable.

Eg:

Resistor, Delay Unit.

✓ If the output of system depends only on future values of input then the system is called Non causal System.

NOTE:

* It cannot be implemented practically. i.e., It can produce its output before input is applied.

Eg:

Image Processing system.

Pblm1: Check whether the following systems are causal or not.

(i) $y(t) = x(t)$ (ii) $y(t) = x(t-1)$

(iii) $y(n) = x(n) + x(n-1)$ (iv) $y(t) = x(t+1)$

(v) $y(n) = x(n-1)$

i) GIVEN:

$$y(t) = x(t) \rightarrow ①$$

TO CHECK:

Causal (or) Not.

SOLUTION:

① Put $t = -1, 0, 1, 2$ in eqn ①.

$\therefore ① \Rightarrow$ $y(-1) = x(-1)$

$y(0) = x(0)$

$y(1) = x(1)$

$y(2) = x(2)$

For all time, output depends on present inputs.

$\therefore$ The given system $y(t) = x(t)$ is a causal system.

(ii) GIVEN:

$$y(t) = x(t-1) \quad \text{—— ①}$$

TO CHECK:

Causal (or) Not

SOLUTION:

Put $t = -1, 0, 1, 2$ in eqn ①

Put $t = -1 \Rightarrow \underset{\text{Present}}{y(-1)} = \underset{\text{Past}}{x(-2)}$

Put $t = 0 \Rightarrow \underset{\text{Present}}{y(0)} = \underset{\text{Past}}{x(-1)}$

Put $t = 1 \Rightarrow \underset{\text{Present}}{y(1)} = \underset{\text{Past}}{x(0)}$

Put $t = 2 \Rightarrow y(2) = x(1)$

For all time, the output of a s/m depends on past values of input.

$\therefore$ The given system $y(t) = x(t-1)$ is a causal system.

(iii) GIVEN:

$$y(n) = x(n) + x(n-1) \quad \text{—— ①}$$

TO CHECK:

Causal (or) Not

SOLUTION:

Put $n = -1, 0, 1, 2$ in eqn ①

Put $n = -1 \Rightarrow \underset{\text{Present}}{y(-1)} = \underset{\text{Present}}{x(-1)} + \underset{\text{Past}}{x(-2)}$

Put $n = 0 \Rightarrow \underset{\text{present}}{y(0)} = \underset{\text{present}}{x(0)} + \underset{\text{Past}}{x(-1)}$

Put $n = 1 \Rightarrow \underset{\text{present}}{y(1)} = \underset{\text{present}}{x(1)} + \underset{\text{Past}}{x(0)}$

Put $n = 2 \Rightarrow \underset{\text{present}}{y(2)} = \underset{\text{present}}{x(2)} + \underset{\text{Past}}{x(1)}$

For all time, the output of a s/m depends on present and past values of input.

$\therefore$ The given system $y(n) = x(n) + x(n-1)$ is a causal system.

(iv) GIVEN:

$$y(t) = x(t+1) \quad \text{—} \; (1)$$

TO CHECK:

Causal (or) Not

SOLUTION:

Put $t = -1, 0, 1$ in eqn (1)

Put $t = -1 \Rightarrow \underset{\text{present}}{y(-1)} = \underset{\text{Future}}{x(0)}$

Put $t = 0 \Rightarrow \underset{\text{present}}{y(0)} = \underset{\text{Future}}{x(1)}$

Put $t = 1 \Rightarrow \underset{\text{present}}{y(1)} = \underset{\text{Future}}{x(2)}$

Put $t = 2 \Rightarrow y(2) = x(3)$

For all time, the output of a s/m depends on future inputs.

$\therefore$ The given system $y(t) = x(t+1)$ is a Non causal system.

STATIC AND DYNAMIC SYSTEM:

✓ If output of system depends only on present values of inputs at each and every instant of time then the system is called STATIC.

✓ Also Known as Memoryless System. Basically, these systems contain no energy storage elements.

Eg: Resistive System.

✓ If output of system depends on past (or) future values of input at any instant of time then the system is called DYNAMIC.

✓ Also Known as Memory System. Basically, these systems contain one (or) more storage elements.

Eg: Capacitive System.

NOTE:

✓ All ~~stati~~ Causal s/ms are Dynamic System. But all dynamic s/ms are need not to be causal s/m.

✓ Integral & Derivative s/ms are Dynamic

✓ All Non causal s/ms are Static System But all Static s/ms are need not to be Non causal s/m.

✓ Time scaling / shifting s/ms are Dynamic

Problem: Check whether the following systems are Static (or) Dynamic.

(i) $y(t) = x(t) + x(t-1)$

(ii) $y(t) = x[\sin t]$

(iii) $y(n) = \text{Even}[x(n)]$

(i) GIVEN:

$$y(t) = x(t) + x(t-1) \quad \text{--- (1)}$$

TO CHECK:

Static (or) Dynamic

SOLUTION:

Put $t = 1, 0, -1$ in eqn (1)

Put $t = 1 \Rightarrow \overset{\text{Pre}}{y(1)} = \overset{\text{Pre}}{x(1)} + \overset{\text{Past}}{x(0)}$

Put $t = 0 \Rightarrow \underset{\text{Pre}}{y(0)} = \underset{\text{Pre}}{x(0)} + \underset{\text{Past}}{x(-1)}$

Put $t = 1 \Rightarrow \underset{\text{Pre}}{y(1)} = \underset{\text{pre}}{x(1)} + \underset{\text{Past}}{x(0)}$

$\because$ The given system $y(t) = x(t) + x(t-1)$ is a Dynamic system. $\because$ Output of a s/m depends on past inputs.

(ii) GIVEN:

$$y(t) = x[\sin t] \quad \text{--- (1)}$$

TO CHECK:

Static (or) Dynamic

SOLUTION:

Put $t = -\pi$ in eqn (1)

$$\therefore (1) \Rightarrow y(-\pi) = x[\sin(-\pi)]$$

$$= x[-\sin\pi]$$

$$\underset{\text{Present}}{y(-3.14)} = \underset{\text{Future}}{x(0)}$$

$\because$ The given system $y(t) = x[\sin t]$ is a Dynamic system. $\because$ Output of a s/m depends on future inputs.

(iii) GIVEN:

$$y(n) = \text{Even}[x(n)] = \frac{x(n) + x(-n)}{2} \quad \text{--- (1)}$$

TO FIND:

Static (or) Dynamic

SOLUTION:

Put $n = 1$ in eqn (1)

$$\therefore (1) \Rightarrow \underset{\text{pre}}{y(1)} = \frac{\overset{\text{pre}}{x(1)} + \overset{\text{past}}{x(-1)}}{2}$$

$\therefore$ The given system $y(n) = \text{Even}[x(n)]$ is a Dynamic system.

STABLE AND UNSTABLE SYSTEM:

✓ If a system satisfies the BIBO (Bounded Input for Bounded Output) stability condition then it is called stable system.

✓ BIBO stability condition is given by,

For DT s/m

$$\sum_{n=-\infty}^{\infty} |h(n)| < \infty$$

For CT s/m

$$\int_{-\infty}^{\infty} |h(t)|\, dt < \infty$$

Where $h(t) / h(n) \rightarrow$ Impulse response of the system.

✓ If a s/m does not satisfy the BIBO condition then it is called Unstable system.

NOTE:

Bounded $\rightarrow$ It's Magnitude is always finite

Problem: Check whether the following systems are stable (or) not.

(i) $y(t) = x(t) + 2$ (ii) $y(n) = n\,x(n)$.

(i) GIVEN:

$$y(t) = x(t) + 2$$

TO CHECK:

Stable (or) Not

SOLUTION:

$$y(t) = x(t) + 2$$

If $x(t) = 10 \Rightarrow y(t) = 10 + 2 = 12$

$\therefore$ The given system $y(t) = x(t) + 2$ is a stable system.

(ii) GIVEN:

$$y(n) = n\,x(n)$$

TO CHECK:

Stable (or) Not

SOLUTION:

$$y(n) = n\,x(n)$$

If $x(n) = 5$

$$\therefore y(n) = 5n < {}^{\infty}_{-\infty}$$

$y(n)$ is not a finite.

$\therefore$ The given system $y(n) = n\,x(n)$ is a unstable s/m.

NOTE:

- The integration and differentiation of signals are Unstable system.

Eg:

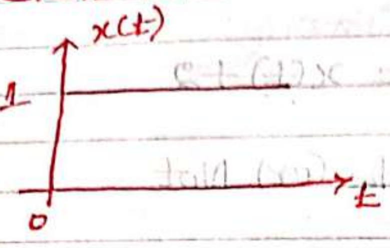
$$y(t) = \frac{d}{dt}\,x(t)$$

$$y(t) = \int x(t)\,dt$$

ANNA UNIVERSITY SOLVED QUESTION.

PART-1

1. Find the even and odd part of the signal.



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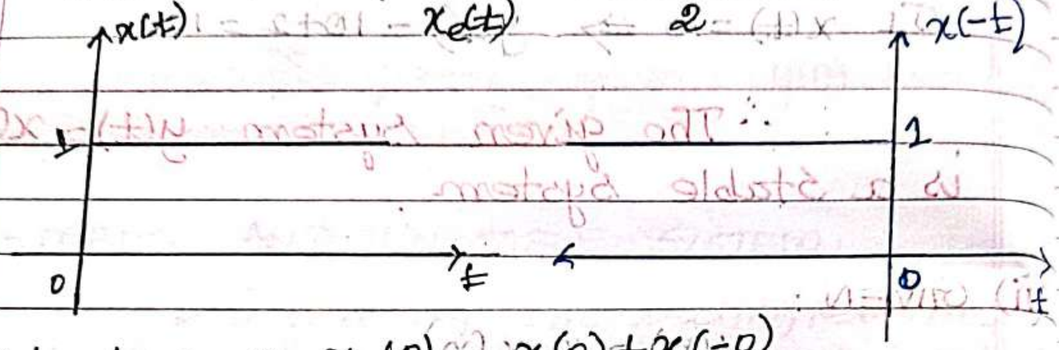
SOLUTION:

W.K.T Even part $= \frac{x(t)+x(-t)}{2}$



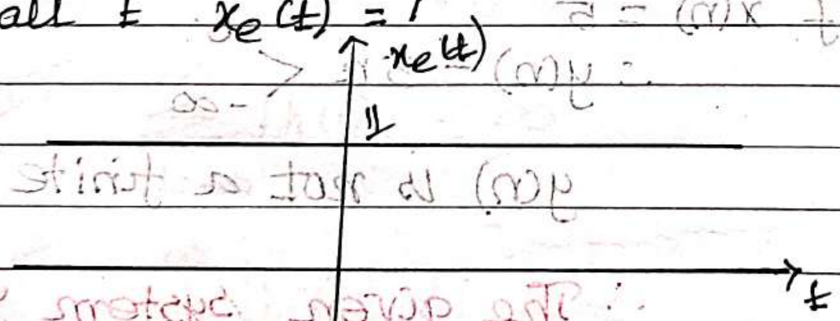
Put $t=0 \Rightarrow x_e(t) = \frac{x(0)+x(-0)}{2}$

$= \frac{1+1}{2} = \frac{2}{2} = 1$

||ly we have

$x_e(1) = x_e(2) = x_e(-1) = x_e(-2) = 1$

For all 't' $x_e(t) = 1$



W.K.T Odd part, $x_o(t) = \frac{x(t)-x(-t)}{2}$

Put $t=0 \Rightarrow x_o(t) = \frac{x(0)-x(-0)}{2} = \frac{1-1}{2} = 0$

||ly wehave $x_o(1) = x_o(-1) = x_o(2) = x_o(-2) = 0$.

For all 't' $x_o(t) = 0$.

2. Determine whether the given DT sequence is periodic or not. If the sequence is periodic, find the fundamental period,
$x(n) = \cos\left(\frac{n}{8}\right)\cos\left(\frac{n\pi}{8}\right)$ APR/MAY 2019.

$\because 2\cos A\cos B = \frac{\cos(A+B) + \cos(A-B)}{2}$

GIVEN:

$$x(n) = \cos(n/8)\cos(n\pi/8) = \frac{\cos\left(\frac{n}{8} + \frac{n\pi}{8}\right) + \cos\left(\frac{n}{8} - \frac{n\pi}{8}\right)}{2}$$

$$= \frac{1}{2}\left\{\cos\left[n\left(\frac{1+\pi}{8}\right)\right] + \cos\left[n\left(\frac{1-\pi}{8}\right)\right]\right\} \quad \text{—— (1)}$$

TO CHECK:

Periodicity.

SOLUTION:

From eqn (1) $\omega_1 = \frac{1+\pi}{8}$; $\omega_2 = \frac{1+\pi}{8}$

W.K.T $N_1 = \frac{2\pi}{\omega_1}$ $N_2 = \frac{2\pi}{\omega_2}$

$$= \frac{2\pi}{\left(\frac{1+\pi}{8}\right)} \qquad N_2 = \frac{2\pi}{\left(\frac{1-\pi}{8}\right)}$$

$$N_1 = \frac{16\pi}{1+\pi} \qquad N_2 = \frac{16\pi}{1-\pi}$$

$$\Rightarrow \frac{N_1}{N_2} = \left(\frac{16\pi}{1+\pi}\right) \Big/ \left(\frac{16\pi}{1-\pi}\right)$$

$$= \left(\frac{\cancel{16\pi}}{1+\pi}\right)\left(\frac{1-\pi}{\cancel{16\pi}}\right)$$

$$\frac{N_1}{N_2} = \frac{1-\pi}{1+\pi}$$

$\frac{N_1}{N_2}$ is a irrational number

$\therefore$ The given DT signal $x(n) = \cos(n/8)\cos(n\pi/8)$ is a non periodic signal.

3. Give the mathematical and graphical representation of DT ramp sequence.

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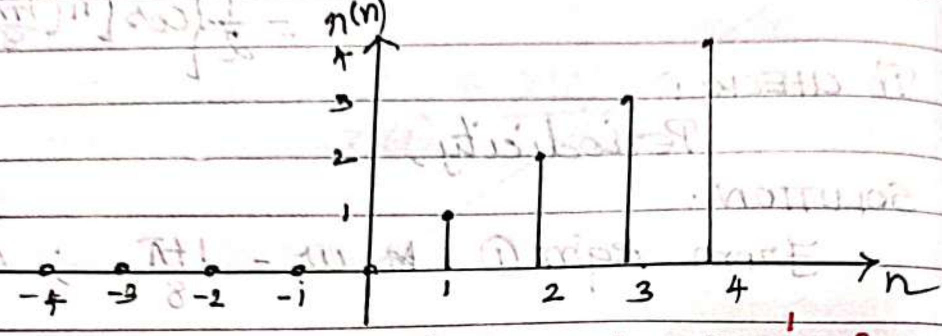
SOLUTION:

Discrete time ramp sequence can be represented as r(n) and defined as,

$$r(n) = \begin{cases} n ; & n \geq 0 \\ 0 ; & n < 0 \end{cases}$$



4. Evaluate the following integral $\int_{-t}^{1} (2t^2+3)\delta(t)\,dt$

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GIVEN:

Let $x(t) = \int_{-t}^{1} (2t^2+3)\,\delta(t)\,dt$.

W.K.T

$$\delta(t) = \begin{cases} 1 ; & t = 0 \\ 0 ; & t \neq 0 \end{cases}$$ ie, Unit Impulse sig.

SOLUTION:

$$\int_{-t}^{1} (2t^2+3)\,\delta(t)\,dt$$

$$= \int_{-t}^{0} \cancel{(2t^2+3)\,\delta(t)}\,dt + \int (2t^2+3)\,\delta(t)\,dt \quad \text{at } t=0$$

$$+ \int_{0}^{1} \cancel{(2t^2+3)\,\delta(t)}\,dt$$

$$= 3 \times 1 = 3$$

$$\therefore \int_{-t}^{1} (2t^2+3)\,\delta(t)\,dt = 3.$$

5. What is random signal? Give an example.

Random signal is one whose occurrence is always random in nature. The pattern of such a signal is quite irregular.

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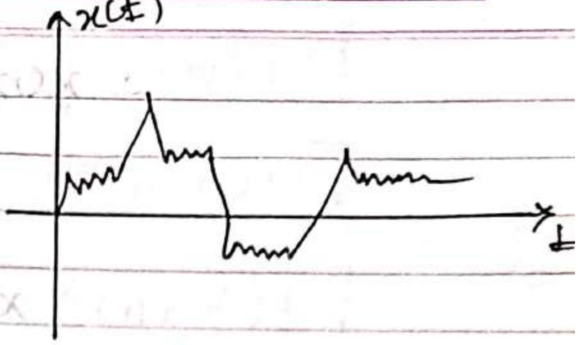
Date: Page:

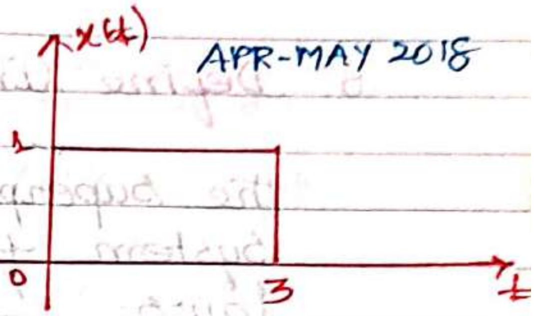
✓ Also known as Non Deterministic signal

Eg:

Thermal noise generated in an electric circuit.



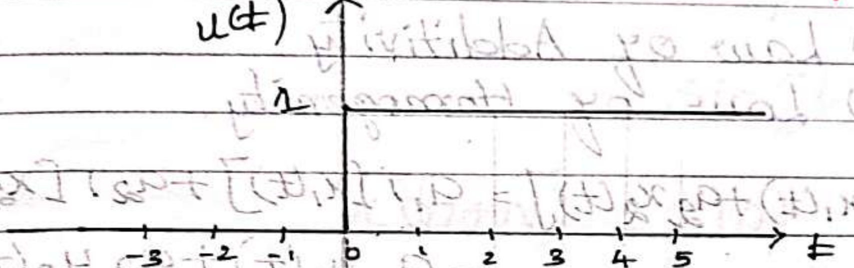
6. Represent given $x(t)$ by $u(t)$ APR-MAY 2018

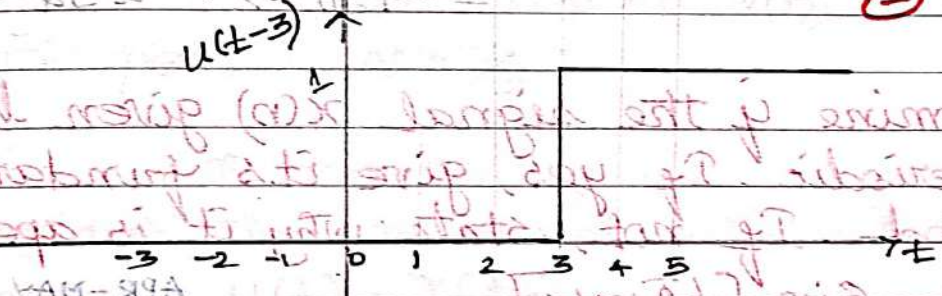


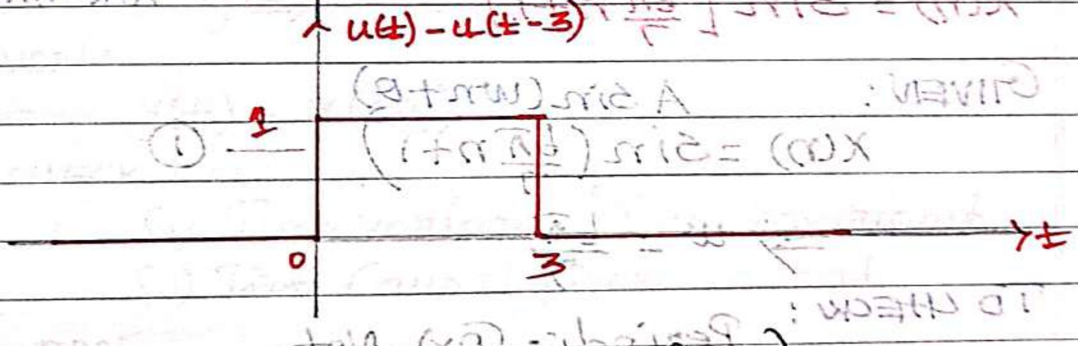
SOLUTION: Unit step signal.

W.K.T $u(t) = 1 ; t \geq 0$

$0 ; t < 0.$







$\therefore x(t) = u(t) - u(t-3)$

7. Find the summation $x(n) = \sum_{n=-\infty}^{\infty} \delta(n-1) \sin 2n$ NOV-DEC2017

SOLUTION:

W.K.T $\delta(n) = 1 ; n = 0$

$0 ; n \neq 0$

$\Rightarrow \delta(n-1) = 1 ; n = 1$

$0 ; n \neq 0.$

$$\therefore x(n) = \sum_{n=-\infty}^{0} \cancel{\delta(n-1)\sin 2n} + \sum_{n=1} \delta(n-1)\sin 2n + \sum_{n=2}^{\infty} \cancel{\delta(n-1)\sin 2n}$$

$$x(n) = \sin 2 = 0.035$$

8. Define Linear System : NOV-DEC 2017

A system is LINEAR if it satisfies the superposition principle. i.e., A linear system follows the combination of two laws.

(i) Law of Additivity

(ii) Law of Homogenity.

$$T[a_1 x_1(t) + a_2 x_2(t)] = a_1 T[x_1(t)] + a_2 T[x_2(t)]$$

$$= a_1 y_1(t) + a_2 y_2(t)$$

9. Determine if the signal x(n) given below is periodic. If yes, give its fundamental period. If not, state why it is aperiodic

$x(n) = \sin\left[\frac{6\pi}{7}n+1\right]$ APR-MAY 2017

GIVEN: $A\sin(\omega n+\theta)$

$$x(n) = \sin\left(\frac{6\pi}{7}n+1\right) \quad \text{--- (1)}$$

$$\Rightarrow \omega = \frac{6\pi}{7}$$

TO CHECK: Periodic (or) Not

SOLUTION:

W.K.T $N = \frac{2\pi}{\omega} = \frac{2\pi}{\left(6\pi/7\right)}$

$$N = \frac{7}{3}$$

W.K.T The condition for periodicity

$$x(n) = x(n+N) \quad \text{--- (2)}$$

Replace 'n' by (n+N) in eqn (1)

$\therefore ① \Rightarrow x(n+N) = \sin\left[\frac{6\pi}{7}(n+N)+1\right]$

$= \sin\left[\frac{6\pi}{7}(n+N)+1\right]$

$= \sin\left[\frac{6\pi}{7}\left(n+\frac{7}{3}\right)+1\right] \quad \because N = 7/3$

$= \sin\left[\left(\frac{6\pi n}{7}+\frac{6\pi}{7}\times\frac{7}{3}\right)+1\right]$

$\boxed{\because \sin(2\pi+\theta) = \sin\theta} \quad = \sin\left[\left(2\pi+\frac{6\pi n}{7}\right)+1\right]$

$= \sin\left[2\pi+\left(\frac{6\pi n}{7}+1\right)\right]$

$= \sin\left[\frac{6\pi n}{7}+1\right]$

$$x(n+N) = x(n)$$

The given signal, $x(n)$ is a periodic with period $N = 7/3$.

10. Check whether the following system is time variant / Time invariant and also causal / non causal. $y(t) = x(t/3)$. APR-MAY 2017.

GIVEN:

$y(t) = x(t/3)$ — ①

TO CHECK:

(i) Time variant / Time Invariant

(ii) ~~Tim~~ Causal / Non causal.

SOLUTION:

Replace 't' by $(t-T)$ in eqn ① on input alone

$\therefore ① \Rightarrow y(t) = x\left(\frac{t-T}{3}\right)$ — ②

Replace 't' by $(t-T)$ in eqn ①

$\therefore ① \Rightarrow y(t-T) = x\left(\frac{t-T}{3}\right)$ — ③

From eqn ② & ③

$$y(t) = y(t-T)$$

$\therefore$ The given $y(t) = x(t/3)$ is Time Invariant system.

Put $t=-1$ in eqn ① $\Rightarrow$ $y(-1) = x(-1/3) = x(-0.33)$
Present Past

Put $t=0$ in eqn ① $\Rightarrow$ $y(0) = x(0)$
Pre Pre

Put $t=1$ in eqn ① $\Rightarrow$ $y(1) = x(1/3) = x(0.33)$
Pre Past

Put $t=2$ in eqn ① $\Rightarrow$ $y(2) = x(2/3) = x(0.66)$

Put $t=3$ in eqn ① $\Rightarrow$ $y(3) = x(3/3) = x(1)$
Pre Past.

$\therefore$ For all time, output of a system at present time depends on present and past values of input.

The given system $y(t) = x(t/3)$ is a Causal system.

11. Define a power signal. APR-MAY 2015

✓ A power signal is one which has finite power and infinite energy. i.e.,

$$0 \leq P \leq \infty \; ; \; E = \infty.$$

$$\text{Power, } P = \lim_{T \to \infty} \frac{1}{2T} \int_{-T}^{T} |x(t)|^2 \, dt$$

12. How the impulse response of DT system is useful in determining its stability and causality? APR-MAY 2015

✓ The stability of DT system is given by,

$$\sum_{n=-\infty}^{\infty} |h(n)| < \infty$$

✓ The causality of DT system is given by

$$h(n) = 0 \; ; \; n < 0.$$

13. Find the value of the integral $\int_{-\infty}^{\infty} e^{-2t} f(t+2)\,dt$

NOV-DEC 2015

GIVEN:

$$\int_{-\infty}^{\infty} e^{-2t} f(t+2)\,dt \quad \text{, Where } f(t) = \text{unit impulse} = \delta(t).$$

$$\Rightarrow f(t+2) = \delta(t+2) = \begin{cases} 1 ; & t = -2 \\ 0 ; & t \neq -2 \end{cases}$$

SOLUTION:

$$\int_{-\infty}^{\infty} e^{-2t} f(t+2)\,dt = \int_{-\infty}^{\infty} e^{-2t}\,\delta(t+2)\,dt$$

$$= e^{-4}$$

$$= 0.018$$

14. Give the relation between continuous time unit impulse function $f(t)$, step function $u(t)$ and ramp function $r(t)$.

NOV-DEC 2015.

SOLUTION:

Let $f(t) = \delta(t) =$ Unit impulse signal.

* $u(t) = \int_{-\infty}^{\infty} \delta(t)\,dt$
* $r(t) = \int_{-\infty}^{\infty} u(t)\,dt$
* $u(t) = \frac{d}{dt} r(t)$
* $\delta(t) = \frac{d}{dt} u(t)$.

15. Determine the value of E and P of the signal. $x(n) = (1/2)^n u(n+1)$

APR-MAY 2014

GIVEN:

$$x(n) = (1/2)^n u(n+1)$$

TO FIND:

$E = ?\ ;\ P = ?$

SOLUTION:

W.K.T Energy, $E = \lim_{N\to\infty} \sum_{n=-N}^{N} |x(n)|^2$

$$= \sum_{n=-\infty}^{\infty} \left|(1/2)^n u(n+1)\right|^2$$

$$= \sum_{n=-\infty}^{\infty} \left(\frac{1}{4}\right)^n [u(n+1)]^2$$

$$= \sum_{n=-1}^{\infty} \left(\frac{1}{4}\right)^n \qquad \because u(n+1) = 1;\ n \geq -1 \quad 0;\ n < -1$$

$$= \left(\frac{1}{4}\right)^{-1} + \sum_{n=0}^{\infty} \left(\frac{1}{4}\right)^n \qquad \because \sum_{n=0}^{\infty} a^n = \frac{1}{1-a}$$

$$= 4 + \frac{1}{1-1/4}$$

$$= 4 + \frac{4}{3} = \frac{16}{3}$$

$$E = 16/3 = \text{Finite}$$

$$\therefore P = 0$$

16. Determine whether or not the given signal $x(t) = \cos(2\pi t + \pi/4)\, u(t+2)$ is periodic if periodic determine the fundamental period. — APR-MAY 2014

GIVEN:

$$x(t) = \cos(\underset{\omega t}{2\pi t} + \pi/4)\, u(t+2) \quad \text{—— (1)}$$

$$\therefore x(t) = \cos(2\pi t + \pi/4) \quad ; t \geq -2$$

$$= 0 \quad ; t < -2$$

TO FIND:

Periodicity, i.e., $x(t) = x(t+T)$

SOLUTION:

From eqn (1) $\omega = 2\pi$

$$\therefore T = \frac{2\pi}{\omega} = \frac{2\pi}{2\pi} = 1$$

$$T = 1$$

Replace 't' by $(t+T)$ in eqn (1):

$$\therefore (1) \Rightarrow x(t+T) = \cos[2\pi(t+T) + \pi/4]\, u(t+2)$$

$$= \cos[2\pi(t+1) + \pi/4] \cos(t+3)$$

$$= \cos[2\pi t + 2\pi + \pi/4] \cos(t+3)$$

$= \cos[2\pi + (2\pi t + \pi/4)]\, u(t+3)$

$= \cos(2\pi t + \pi/4)\, u(t+3)$

$x(t+T) = x(t)$

$\therefore$ The given signal $x(t)$ is a PERIODIC signal with period $T = 1$ sec

17. Check whether the discrete time signal $\sin 3n$ is periodic. APR-MAY 2013

GIVEN:

Let $x(n) = \sin 3n$ — (1)

TO CHECK:

Periodicity

SOLUTION:

From eqn (1) $\Omega = 3$

$\therefore N = \frac{2\pi}{\Omega} = \frac{2\pi}{3}$ (~~= Irrational~~)

Replace 'n' by $(n+N)$ in eqn (1)

$\therefore (1) \Rightarrow x(n+N) = \sin 3(n+N)$

$= \sin 3\left(n + \frac{2\pi}{3}\right)$

$= \sin\left[3n + 3 \times \frac{2\pi}{3}\right]$

$= \sin[2\pi + 3n]$

$= \sin 3n$

$x(n+N) = x(n)$

$\therefore$ The given signal $\sin 3n$ is a periodic with period $N = 2\pi/3$

APR-MAY 2013

18. Define Random signal.

Refer Qn. No: 5 (APR-MAY 2018)

19. Evaluate $\int_{-\infty}^{\infty}(2t^2+3)\,\delta(t)\,dt$ NOV/DEC 2013

Refer Qn. No: 4 (NOV/DEC 2018)

20. Determine the fundamental time period of the signal $x(t) = \frac{3}{5}\cos(4t+\pi/3) + 8/3\,\sin(8t+\pi/2)$

GIVEN:

$$x(t) = \frac{3}{5}\cos(4t+\pi/3) + 8/3\,\sin(8t+\pi/2) \quad \text{---} (1)$$

TO FIND:

Fundamental time period, T

SOLUTION:

From eqn (1)

$$\omega_1 = 4 \qquad ; \omega_2 = 8$$

$$\Rightarrow T_1 = \frac{2\pi}{\omega_1} \qquad \Rightarrow T_2 = \frac{2\pi}{\omega_2}$$

$$= \frac{2\pi}{4} \qquad T_2 = \frac{2\pi}{8}$$

$$T_1 = \pi/2 \qquad T_2 = \pi/4$$

$$\therefore \frac{T_1}{T_2} = \frac{(\pi/2)}{(\pi/4)} = \frac{\pi}{2} \times \frac{4}{\pi}$$

$$\frac{T_1}{T_2} = 2 \text{ (Rational Number)} \quad \text{---} (2)$$

∴ Periodic signal,

From (2) $T_1 = 2T_2$

$$T_1 = 2 \times \pi/4$$

$$T_1 = \pi/2$$

The fundamental period $T = \pi/2$

PART-B

1.a) i) Draw the waveforms represented by the following step functions.

$$f_1(t) = 2u(t-1) \quad ; \quad f_2(t) = -2u(t-2)$$

$$f_3(t) = f_1(t) + f_2(t) \quad ; \quad f_4(t) = f_1(t) - f_2(t)$$

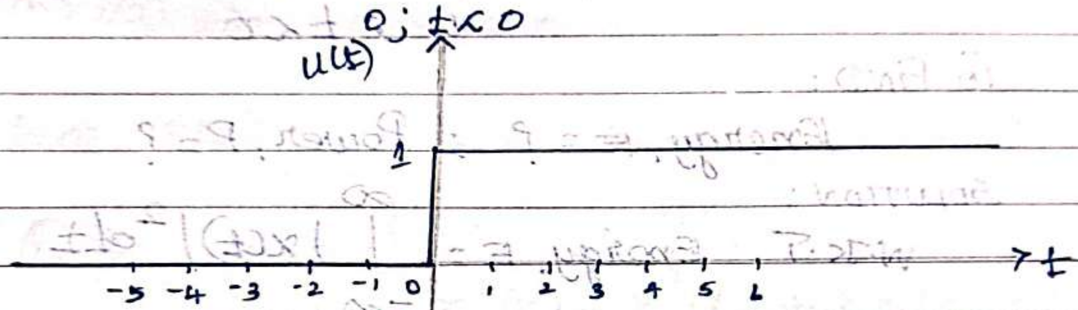
ii) Determine the energy and power of the given signal $x(t) = t\,u(t)$.

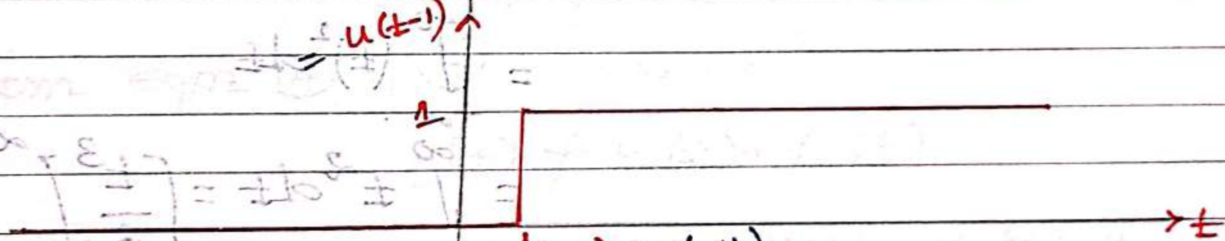
iii) Check whether the given system is linear (or) not $y(t) = x^2(t)$. APR/MAY 2019.

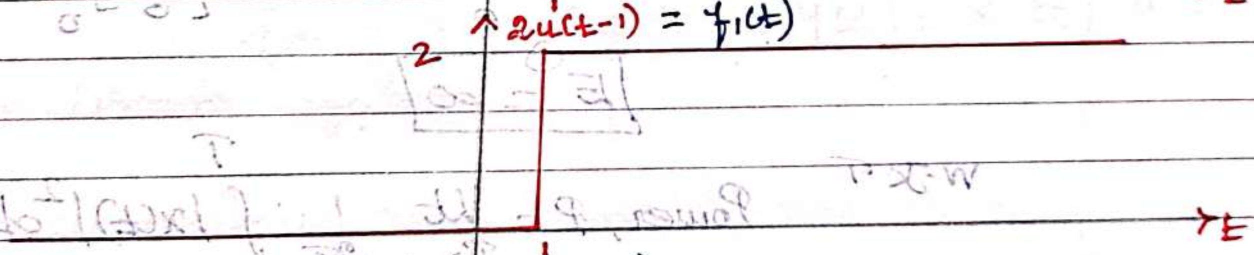
i) SOLUTION:

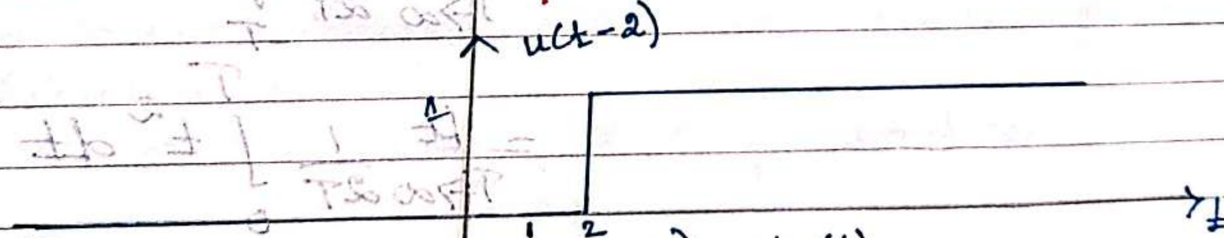
W.K.T $u(t) = 1 ; t \geq 0$

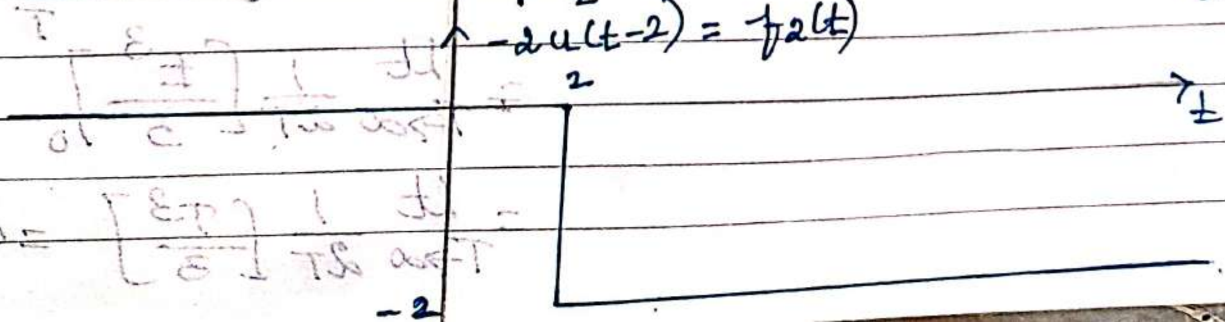
$\qquad\quad\; = 0 ; t < 0$

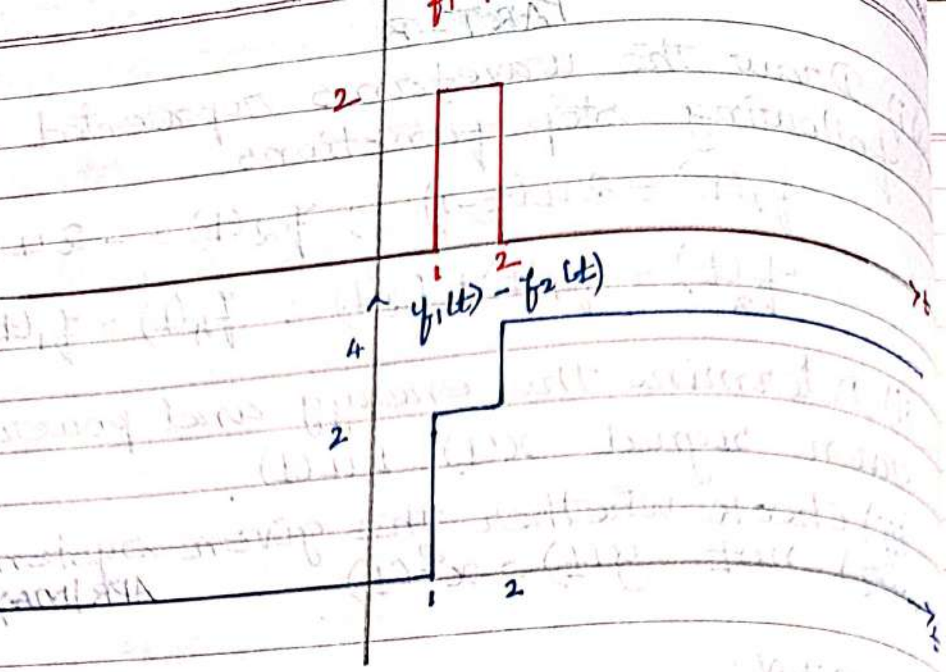












ii) GIVEN:

$$x(t) = t\,u(t) = \begin{cases} t \,; & t \geq 0 \\ 0 \,; & t < 0 \end{cases}$$

TO FIND:

Energy, $E = ?$; Power, $P = ?$

SOLUTION:

W.K.T, Energy, $E = \int_{-\infty}^{\infty} |x(t)|^2 dt$

$$= \int_{-\infty}^{\infty} (t)^2 dt$$

$$= \int_{0}^{\infty} t^2 dt = \left[\frac{t^3}{3}\right]_0^{\infty}$$

$$\boxed{E = \infty}$$

W.K.T

$$\text{Power, } P = \lim_{T \to \infty} \frac{1}{2T} \int_{-T}^{T} |x(t)|^2 dt$$

$$= \lim_{T \to \infty} \frac{1}{2T} \int_{0}^{T} t^2 dt$$

$$= \lim_{T \to \infty} \frac{1}{2T} \left[\frac{t^3}{3}\right]_0^{T}$$

$$= \lim_{T \to \infty} \frac{1}{2T} \left[\frac{T^3}{3}\right] = 0$$

$$\boxed{\begin{array}{c} \text{Energy}, E = \infty \\ \hline \text{Power}, P = 0 \end{array}}$$

$\therefore$ The given signal $x(t) = tu(t)$ is an either energy nor power signal.

iii) GIVEN:

$$y(t) = x^2(t) = [x(t)]^2 \quad \text{——} (1)$$

TO CHECK:

Linearity

SOLUTION:

From eqn (1) $\quad y_1(t) = x_1^2(t) \quad - (2)$

$\qquad\qquad\qquad y_2(t) = x_2^2(t) \quad - (3)$

$(2) + (3) \Rightarrow y_1(t) + y_2(t) = x_1^2(t) + x_2^2(t)$

$\qquad\qquad\qquad\qquad = [x_1(t)]^2 + [x_2(t)]^2 \quad \text{——} (4)$

Replace $x(t)$ by $x_1(t) + x_2(t)$ in eqn (1)

$\therefore (1) \Rightarrow \quad y(t) = [x_1(t) + x_2(t)]^2$

$\qquad\qquad y(t) = x_1^2(t) + x_2^2(t) + 2x_1(t)x_2(t) \quad - (5)$

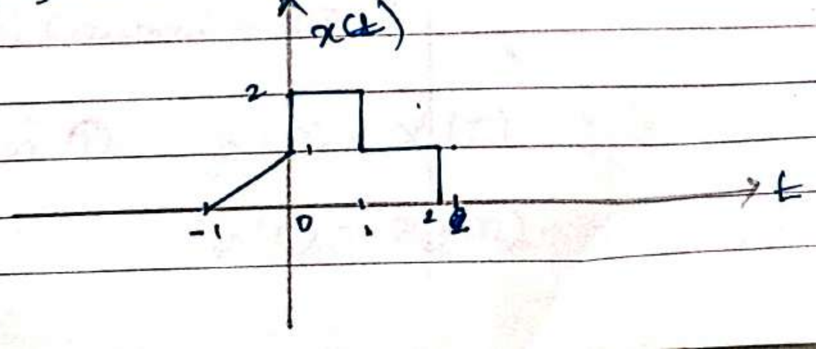
From eqn (4) & (5)

$$y(t) \neq y_1(t) + y_2(t).$$

$\therefore$ The given system $y(t) = x^2(t)$ is a Non-Linear system.

b) i) A continuous time signal $x(t)$ is shown in fig. below, sketch and label each of the following signal

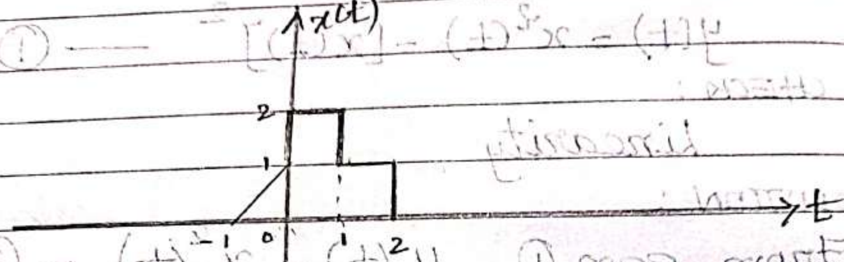
$x(t-2)$; $x(2t+3)$; $x(\frac{3}{2}t)$ and $x(-t+1)$

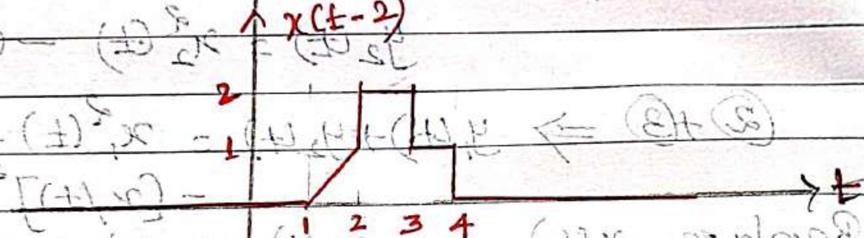


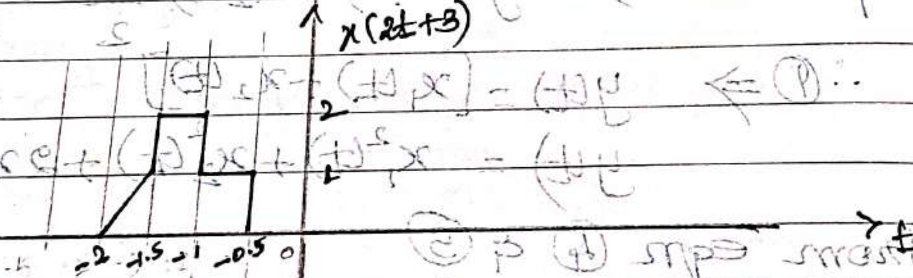
ii) Determine the energy and power of the signal $x(n) = \cos(\frac{\pi}{4} n)$

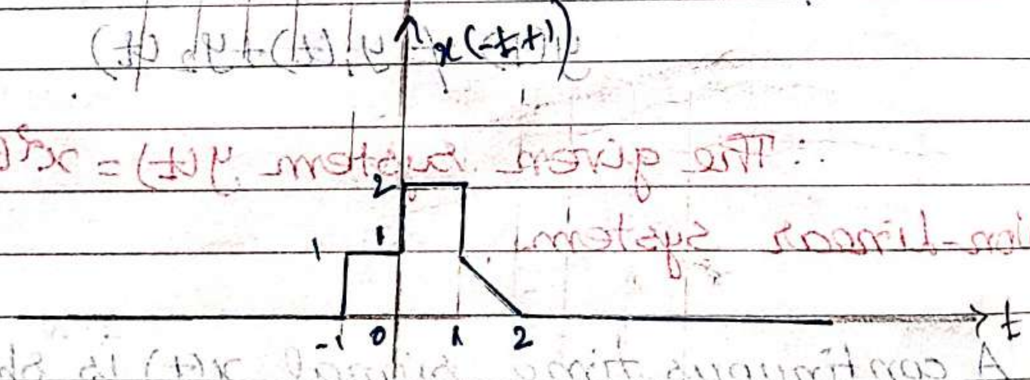
iii) Check whether the given system is linear/nonlinear, Time variant/Time invariant, Causal/Non causal $y(n) = x(n) - x(n-1)$

b) i) SOLUTION:









ii) GIVEN:

$$x(n) = \cos(\pi/4\, n)$$

TO FIND

Energy, E = ? ; Power, P = ?

SOLUTION:

W.K.T $\quad$ Energy, $E = \sum_{n=-\infty}^{\infty} |x(n)|^2$

$$= \sum_{n=-\infty}^{\infty} |\cos(\pi/4) n|^2$$

$\therefore \cos^2\theta = \frac{1+\cos 2\theta}{2}$

$$= \sum_{n=-\infty}^{\infty} \cos^2\left(\frac{n\pi}{4}\right)$$

$$= \sum_{n=-\infty}^{\infty} \frac{1+\cos\left(2\times\frac{n\pi}{4}\right)}{2}$$

$$= \sum_{n=-\infty}^{\infty} \left[\frac{1}{2} + \frac{\cos(n\pi/2)}{2}\right]$$

$\therefore \sum_{n=-\infty}^{\infty} \cos(n\pi/2) = 1$

$$= \frac{1}{2} + \sum_{n=-\infty}^{\infty} \frac{\cos(n\pi/2)}{2}$$

$$E = \frac{1}{2} + \frac{1}{2}$$

$$\boxed{E = 1}$$

$$\therefore \boxed{P = 0}$$

(iii) GIVEN:

$$y(n) = x(n) - x(n-1) \quad \text{—} \ (1)$$

TO CHECK:

(i) Linear (or) Not

(ii) Time variant (or) Not

(iii) Causal (or) Not

SOLUTION:

From eqn (1) $\quad y_1(n) = x_1(n) - x_1(n-1) \quad$ — (2)

$$y_2(n) = x_2(n) - x_2(n-1) \quad \text{—} \ (3)$$

② + ③ $\Rightarrow y_1(n) + y_2(n) = x_1(n) - x_1(n-1) + x_2(n) - x_2(n-1)$ — ④

Replace 'x' by $x_1 + x_2$ in eqn ①

$\therefore$ ① $\Rightarrow$ $y(n) = x_1(n) - x_1(n-1) + x_2(n) - x_2(n-1)$ — ⑤

From eqns ④ & ⑤

$$y(n) = y_1(n) + y_2(n)$$

$\therefore$ The given system $y(n) = x(n) - x(n-1)$ is a linear system

Replace 'n' by $(n-n_0)$ on input in eqn ①

$\therefore$ ① $\Rightarrow$ $y(n) = x(n-n_0) - x_1[(n-n_0)-1]$ — ⑥

Replace 'n' by $(n-n_0)$ in eqn ①

$\therefore$ ① $\Rightarrow$ $y(n-n_0) = x(n-n_0) - x[(n-n_0)-1]$ — ⑦

From eqns ⑥ & ⑦

$$y(n) = y(n-n_0)$$

$\therefore$ The given system $y(n) = x(n) - x(n-1)$ is a Time Invariant system

Put $n = -2, -1, 0, 1, 2$ in eqn ①

$\underbrace{y(-2)}_{\text{Pre}} = \underbrace{x(-2)}_{\text{Pre}} + \underbrace{x(-3)}_{\text{Past}}$

$\underbrace{y(-1)}_{\text{Pre}} = \underbrace{x(-1)}_{\text{Pre}} + \underbrace{x(-2)}_{\text{Past}}$

$\underbrace{y(0)}_{\text{Pre}} = \underbrace{x(0)}_{\text{Pre}} + \underbrace{x(-1)}_{\text{Past}}$

$\underbrace{y(1)}_{\text{Pre}} = \underbrace{x(1)}_{\text{Pre}} + \underbrace{x(0)}_{\text{Past}}$

$\underbrace{y(2)}_{\text{Pre}} = \underbrace{x(2)}_{\text{Pre}} + \underbrace{x(1)}_{\text{Past}}$

$\therefore$ The given system is Causal system.